

**LECTURE NOTES ON  
THEORY OF MACHINES  
DIPLOMA 4<sup>th</sup> SEM  
MECHANICAL BRANCH**

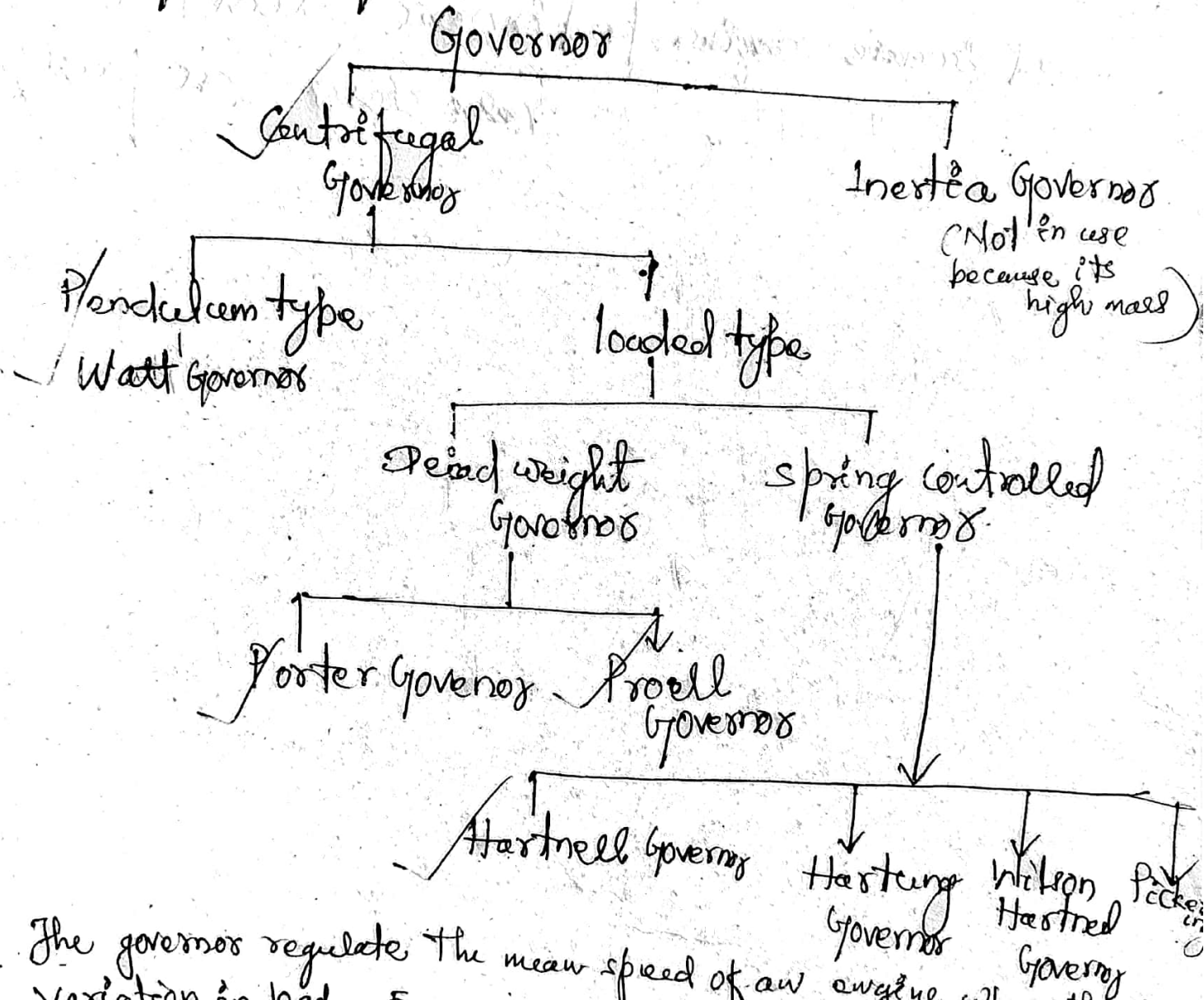
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# GOVERNOR

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The function of the governor is to regulate the mean speed of an engine, when there is variation of load.

## Classification of Governors

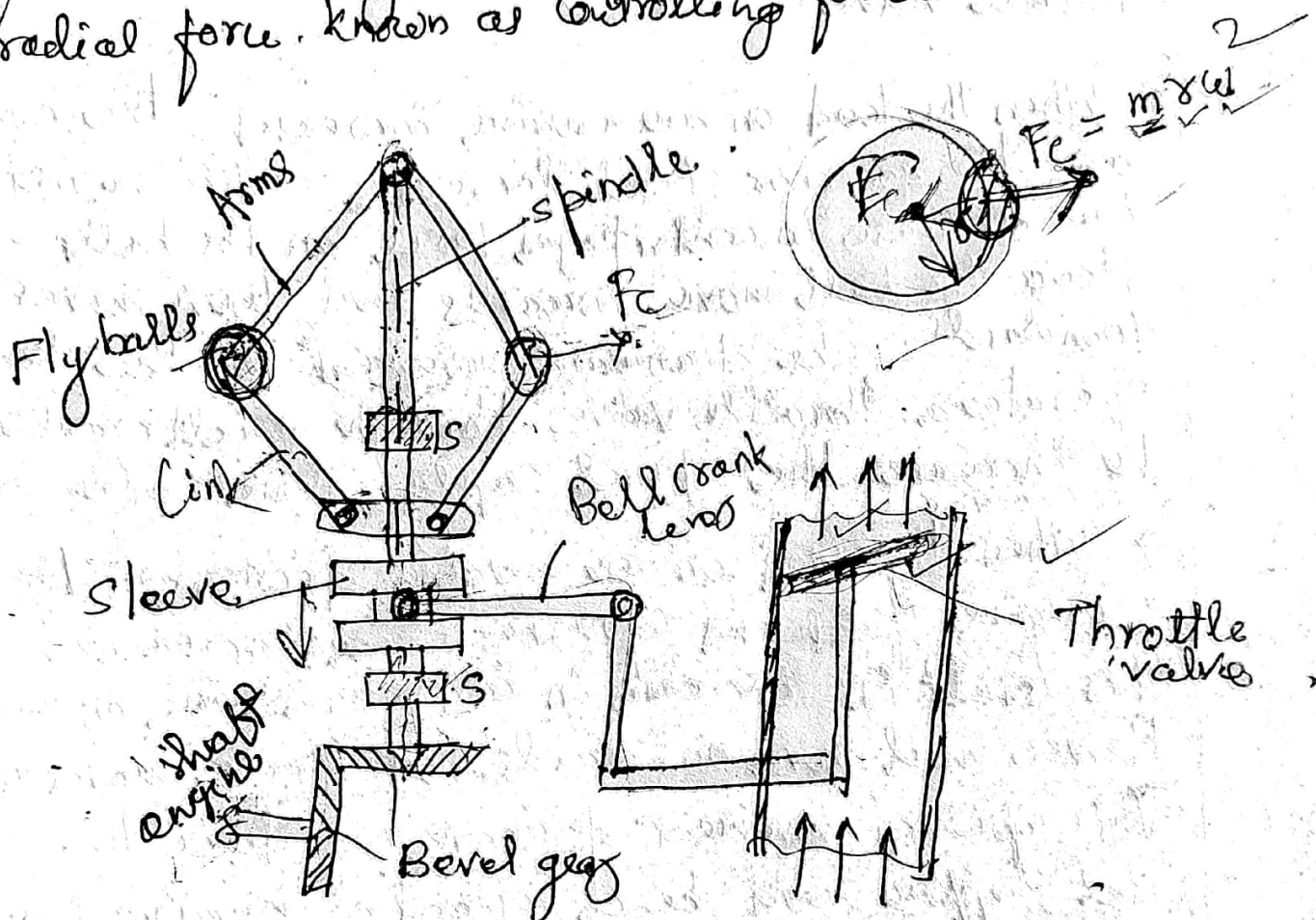


The governor regulate the mean speed of an engine where there is variation in load. For ex. when load on an engine increases its speed decreases, therefore it is necessary to supply more working fluid.

On the other hand if the load on an engine decreases its speed increases and hence less working fluid is required. The Governor automatically control the supply of working fluid to the engine with different load cond<sup>n</sup> and keep the mean speed within a range.

## Centrifugal Governor

Centrifugal Governor are based on balancing of Centrifugal force on rotating balls by an equal and opposite radial force. known as controlling force.



- The centrifugal governor has two balls having equal mass known as Fly balls which are attached to the arms as shown in fig. The balls revolve with the spindle
- The spindle is driven by engine through bevel gear.
  - The upper end of the arms are pivoted to the spindle so that the balls may rise up and down as they revolve about spindle axis.
  - The arms are connected to the sleeve by a link
  - The sleeve is keyed to the spindle, but slide up and down
  - The sleeve rotates with the spindle, but can slide up and down
  - The Governor bell rises and sleeve rises up when engine speed increases and falls down when speed decreases

limit the  
In order to, travel of sleeve in upward and downward directions, two stops are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve.

→ When the load on an engine increases, the engine and the governor speed decreases. This result in decrease in centrifugal force on the balls. Hence the ball move inwards and sleeve moves downwards. The downward movement of the sleeve operate a throttle valve, through a bell crank lever to increase the fuel supply. Hence engine speed increases.

→ When the load on an engine decreases, the engine speed and governor speed increases. This result in increase in centrifugal force on the balls. Hence ball moves outwards and sleeve moves upward. This upward movement of sleeve reduce the fuel supply and hence speed of engine is decreased.

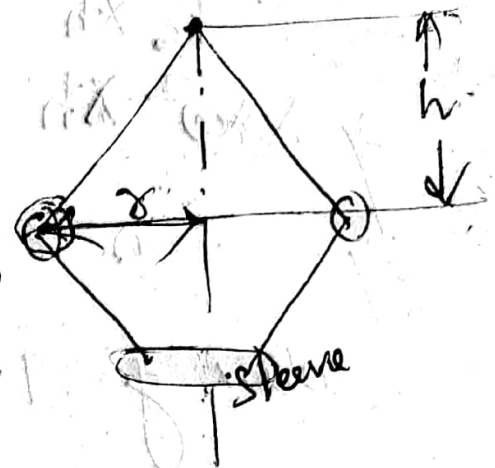
In this way the centrifugal governor controls the speed of an engine.



# Terms Used In Governor

## Height of Governor (h)

It is the vertical distance from the center of balls to the point where the arms cross the spindle.



## Equilibrium speed

It is speed at which the governor balls, arms etc are in complete equilibrium and the sleeve does not move up and downwards.

## Maximum equilibrium speed

## Minimum equilibrium speed

→ The speed at the maximum radius of rotation of balls without tending to move upwards and down called Maximum equilibrium speed.

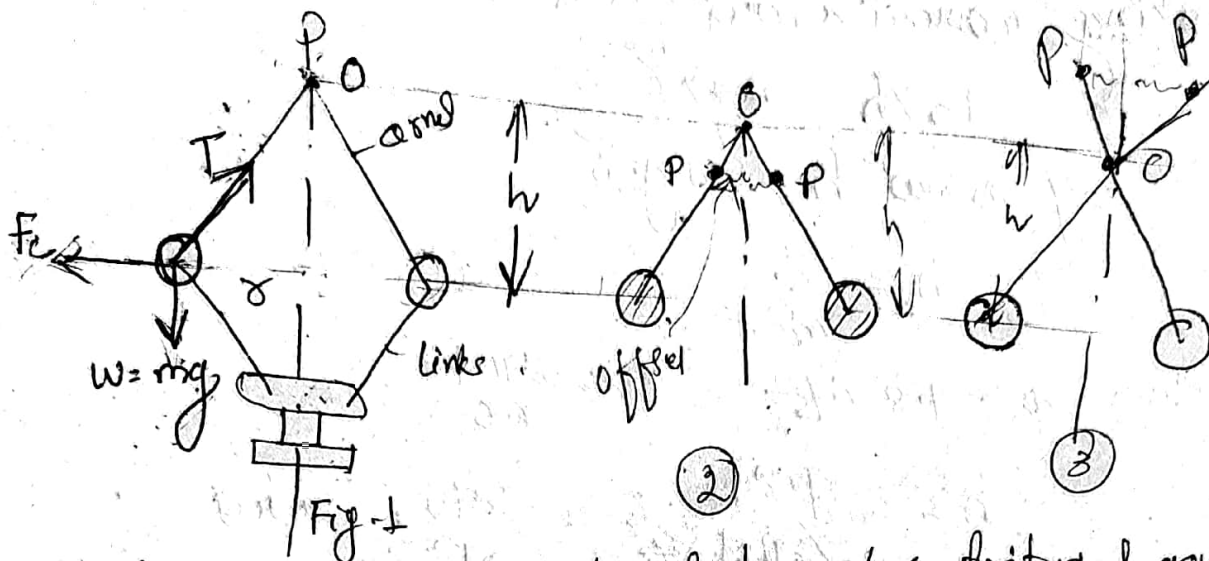
→ The speed at the minimum radius of rotation of balls without tending to move upwards and downward is called minimum equilibrium speed.

## sleeve lift.

It is the vertical distance which the sleeve travels due to change in equilibrium speed.

# Watt Governor

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Watt Governor is a simplest form of centrifugal governor  
 → It is basically a conical pendulum with link attached to a sleeve of negligible mass  
 → The arms of Governor are connected to the spindle in three different ways.

1. The pivot P, may be on the spindle axis (Fig 1)
2. The pivot P may be offset from spindle axis and arms produced intersect at O.
3. The pivot P may be offset, but the arms cross the axis at O.

Let  $m$  = mass of the ball in kg  
 $T$  = Tension in the arm  
 $\omega$  = angular velocity of arm and ball  
 $= \frac{2\pi N}{60}$

$r$  = radius of rotation of ball  
 $F_c$  = Centrifugal force =  $m r \omega^2$   
 $h$  = height of Governor.

It is assumed that weight of arms, links and sleeve are negligible as compared to weight of ball  
 Now the ball is in equilibrium under the action  
 of (1)  $F_c$  (2)  $T$  (3)  $w = mg$

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Taking moment about point O we have

$$F_c \times h = m g \times r$$

$$\Rightarrow m \times \omega^2 h = m g r$$

$$\Rightarrow h = \frac{g}{\omega^2}$$

when  $g = 9.8 \text{ m/s}^2$        $\omega = \frac{2\pi N}{60}$

$$h = \frac{9.81}{\left(\frac{2\pi N}{60}\right)^2} = \frac{895}{N^2} \text{ metres}$$

$$\boxed{h = \frac{895}{N^2}} \quad \checkmark$$

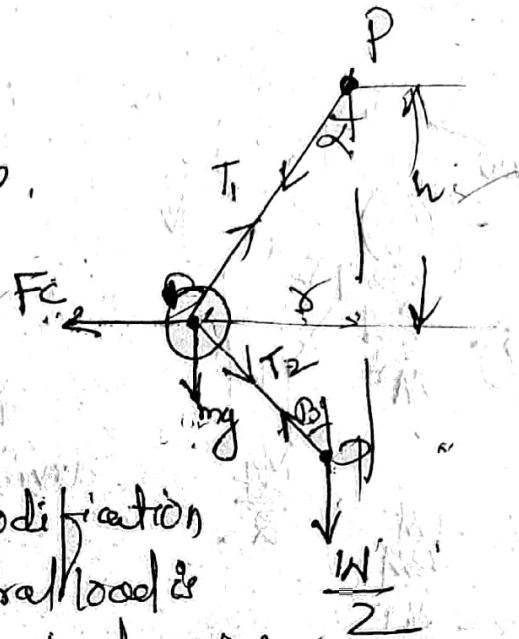
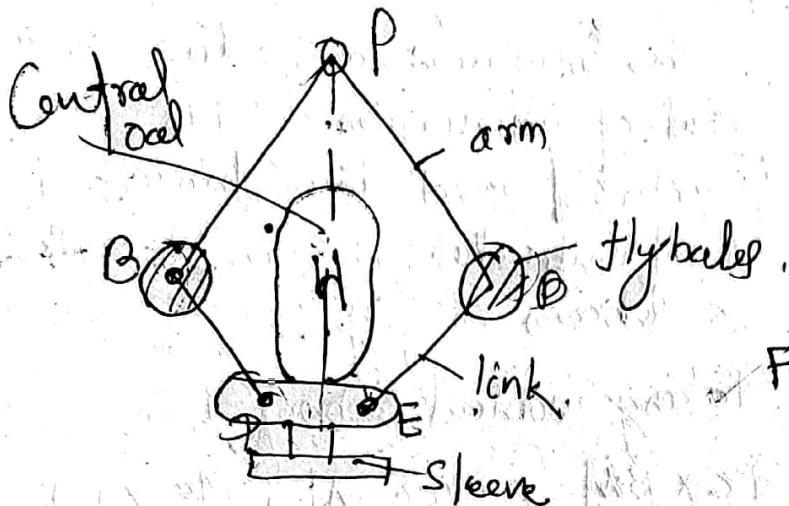
$$\boxed{N^2 = \frac{895}{h}}$$

From the above expression it is observed that the height of governor  $h$ , is inversely proportional to  $N^2$ . Hence at high speed the value of  $h$  is small. At such speed the change in the value of  $h$  corresponding to small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply.

→ This Governor is only work satisfactorily at low speed (60 to 80 rpm).

# Porter Governor

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The Porter governor is a modification of watt governor with a central load is placed on the sleeve. The load moves up and down with the sleeve.

Let  $m$  = mass of fly ball in kg

$W$  = weight of central load =  $Mg$

$\omega$  = Angular speed of balls in rad/sec =  $\frac{2\pi N}{60}$

$F_c$  = centrifugal force on balls =  $m\omega^2 r$

$T_1$  = Tension on the arm

$T_2$  = Force on the link

$\alpha$  = Angle of inclination of the arm with vertical

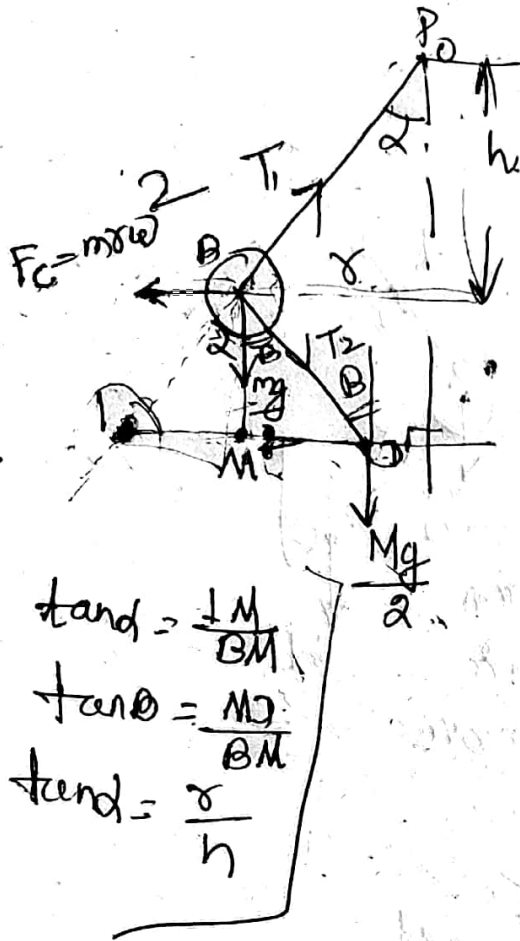
$\beta$  = Angle of inclination of the link to the vertical.

There are two methods to determine the height of governor  $h$  and angular speed  $\omega$

Method-1 Resolution of forces

Method-2 Instantaneous center method

Instantaneous Center Method . Consider equilibrium of  $\Delta P$  (8)



The instantaneous center lies at a point of intersection of PB line produced and line through P perpendicular to spindle axis as shown in fig.

Taking moment about I

$$F_c \times BM = mg \times IM + \frac{Mg}{2} \times IP$$

divide BM both side.

$$F_c = mg \frac{IM}{BM} + \frac{Mg}{2} \frac{IP}{BM}$$

$$= mg \frac{IM}{BM} + \frac{Mg}{2} \left( \frac{IM + IP}{BM} \right)$$

$$= mg \tan \alpha + \frac{Mg}{2} (\tan \alpha + \tan \beta)$$

$$= mg \tan \alpha + \frac{Mg}{2} \tan \alpha \left( 1 + \frac{\tan \beta}{\tan \alpha} \right)$$

$$= \tan \alpha \left( mg + \frac{Mg}{2} \left( 1 + \frac{\tan \beta}{\tan \alpha} \right) \right)$$

$$= \tan \alpha \left( mg + \frac{Mg}{2} (1 + q) \right)$$

Dividing  $\tan \alpha$  throughout

$$\frac{F_c}{\tan \alpha} = mg + \frac{Mg}{2} (1 + q)$$

$$\left[ \because q = \frac{\tan \beta}{\tan \alpha} \right]$$

We know  $F_c = m r \omega^2$  and  $\tan \alpha = \frac{r}{h}$  ✓

$$\therefore \frac{m r \omega^2}{r/h} = mg + \frac{Mg}{2} (1 + q)$$

$$\Rightarrow m \omega^2 h = mg + \frac{Mg}{2} (1 + q)$$

$$\Rightarrow h = \frac{mg + \frac{Mg}{2} (1 + q)}{m \omega^2} \times \frac{1}{\omega^2}$$

$$= \frac{m + \frac{M}{2} (1 + q)}{\omega^2} g$$



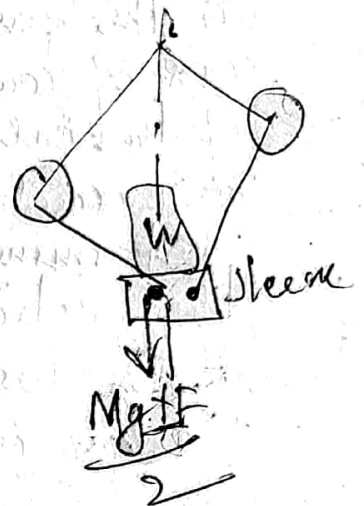
$$h = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

(9)

$$\Rightarrow \omega^2 = \frac{m + \frac{M}{2}(1+q)}{m} \frac{g}{h}$$

$$\Rightarrow \left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \frac{g}{h}$$

$$\Rightarrow N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$



Note-1

when  $\alpha = 0$   $\tan \alpha = \tan 0$  Hence  $q = \frac{\tan \theta}{\tan \alpha} = 1$

$$N^2 = \frac{m + \frac{M}{2}(1+1)}{m} \frac{895}{h} = \left(\frac{m+M}{m}\right) \frac{895}{h}$$

Note-2 when the loaded sleeve move up and down the frictional force act in a direction opposite to the movement of sleeve

$$N^2 = \frac{mg + \frac{Mg \pm F}{2}(1+q)}{mg} \times \frac{895}{h}$$

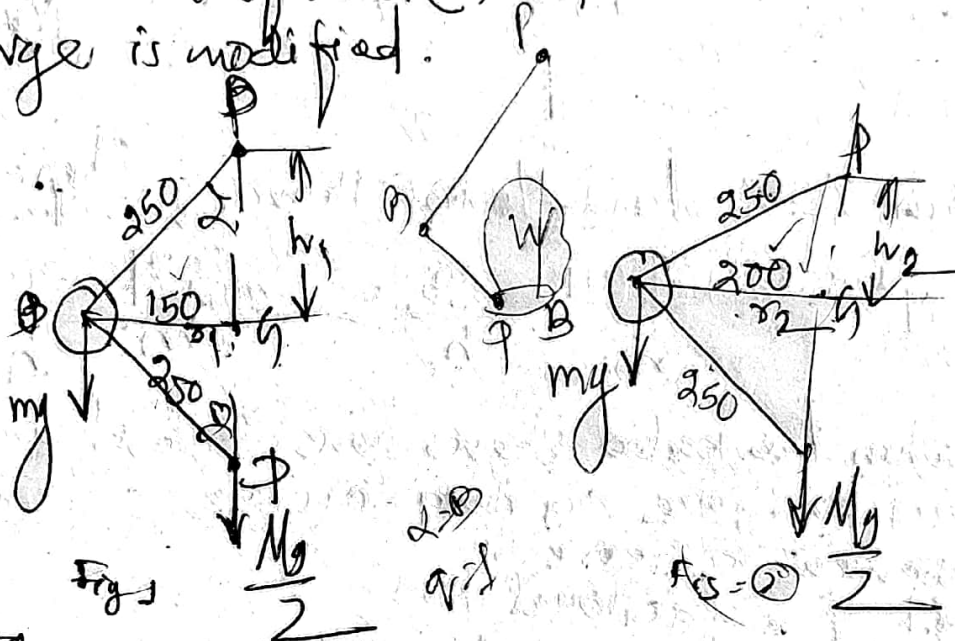
when  $q=1$

$$N^2 = \frac{mg + Mg \pm F}{mg} \times \frac{895}{h}$$

# Problems on Porter Governor

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- The arms of Porter governor are each 250 mm and pivoted on the governor axis. The mass of each ball is 5 kg and mass of central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of governor if the friction at the sleeve is equivalent of 20 N of the load of sleeve, determine how the speed range is modified.



Given data

$$BP = B\phi = 250 \text{ mm} \quad m = 5 \text{ kg} \quad M = 30 \text{ kg} \quad r_1 = 150 \text{ mm} \quad r_2 = 200 \text{ mm}$$

Let  $N_1$  = Minimum speed at  $r_1 = 150 \text{ mm}$

$N_2$  = Maximum speed at  $r_2 = 200 \text{ mm}$

Fig 1  $h_1 = \sqrt{PB^2 - r_1^2} = \sqrt{250^2 - 150^2} = 200 \text{ mm} = 0.2 \text{ m}$

$$N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+30}{5} \times \frac{895}{0.2} = 31325$$

$$N_1 = 177 \text{ rpm}$$

Fig 2  $h_2 = \sqrt{250^2 - 200^2} = 150 \text{ mm} = 0.15 \text{ m}$

We know that  $N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+30}{5} \times \frac{895}{0.15} = 41767$

$$N_2 = 204.4 \text{ rpm}$$

Range of speed of governor =  $N_2 - N_1 = 204.4 - 177 = 27.4 \text{ rpm}$

Speed range of Governor when frictional force on sleeve = 20 N

when sleeve moves down  
 $F \uparrow$

$$N_1^2 = \frac{mg + (Mg - F)}{m \cdot g} \times \frac{895}{h_1}$$

$$N_1^2 = \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29500$$

$$N_1 = 172 \text{ rpm}$$

when sleeve moves upward  
 Frictional force is down  
 $F \downarrow$

$$N_2^2 = \frac{mg + (Mg + F)}{m \cdot g} \times \frac{895}{h_2}$$

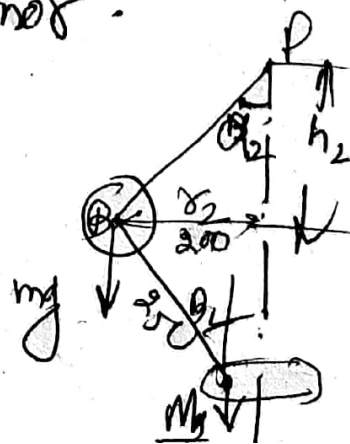
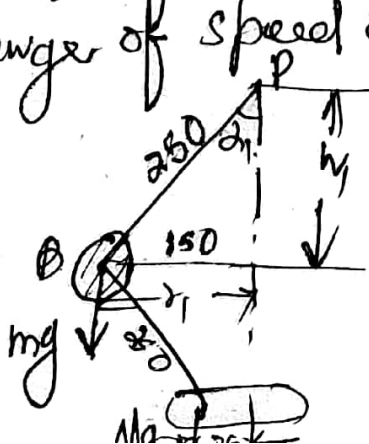
$$N_2 = \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44200$$

$$N_2 = 210 \text{ rpm}$$

Speed range of Governor =  $N_2 - N_1 = 210 - 172 = 38 \text{ rpm}$

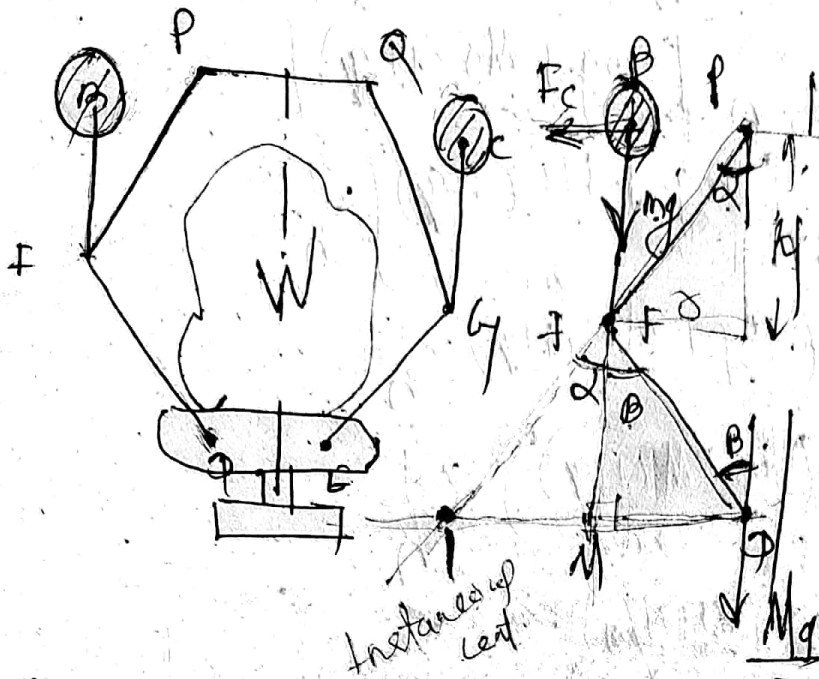
H.W

A porter Governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of Governor.



# PROELL GOVERNOR

(12)



$$\tan \alpha = \frac{x}{h}$$

$$\tan \alpha = \frac{IM}{FM}$$

$$\tan \theta = \frac{MD}{FM}$$

The Proell governor has balls fixed at B and C to the extensions of link FD and EQ respectively. The arm FP is pivoted at P and arm GQ is pivoted at Q. Consider the equilibrium of half of governor as shown in fig.

The instantaneous center lies at the intersection of the PF line produced and line drawn from D which is  $\perp$  to the spindle axis. Now  $BM \perp ID$ .

Taking moment about I

$$F_c \times BM = mg \times IM + \frac{Mg}{2} \times ID$$

$$\Rightarrow F_c \times \frac{BM}{BM} = mg \times \frac{IM}{BM} + \frac{Mg}{2} \times \left( \frac{IM + MD}{BM} \right)$$

$$\Rightarrow F_c = \frac{FM}{BM} \left( \frac{IM}{FM} + \frac{Mg}{2} \left( \frac{IM}{FM} + \frac{MD}{FM} \right) \right)$$

$$= \frac{FM}{BM} \left( mg \times \frac{IM}{FM} + \frac{Mg}{2} \left( \frac{IM}{FM} + \frac{MD}{FM} \right) \right)$$

$$= \frac{FM}{BM} \left( mg \times \tan \alpha + \frac{Mg}{2} (\tan \alpha + \tan \theta) \right)$$

$$= \frac{FM}{BM} \left( mg \times \tan \alpha + \frac{Mg}{2} \tan \alpha (1 + \frac{\tan \theta}{\tan \alpha}) \right)$$

$$= \frac{FM}{BM} \tan \alpha \left( mg + \frac{Mg}{2} (1 + \frac{\tan \theta}{\tan \alpha}) \right)$$

We know  $F_c = m r \omega^2 \tan \alpha = \frac{x}{h}$ ,  $q = \frac{\tan \beta}{\tan \alpha}$

$$\Rightarrow m r \omega^2 = \frac{F M}{B M} \times \frac{x}{h} \left[ m g + \frac{M g}{2} (1+q) \right]$$

$$\Rightarrow \omega^2 = \frac{F M}{B M} \left[ \frac{m g + \frac{M g}{2} (1+q)}{m} \right] \times \frac{1}{h}$$

$$\Rightarrow \omega^2 = \frac{F M}{B M} \left( \frac{m g + \frac{M g}{2} (1+q)}{m} \right) \times \frac{1}{h}$$

$$= \frac{F M}{B M} \left( \frac{m + \frac{M}{2} (1+q)}{m} \right) \times \frac{g}{h}$$

and  $\omega = \frac{2000}{60}$  and  $g = 9.81 \text{ N/m}^2$

$$N^2 = \frac{F M}{B M} \left( \frac{m + \frac{M}{2} (1+q)}{m} \right) \times \frac{895}{h}$$

Note: if  $\alpha = \beta$   $\tan \alpha = \tan \beta$   
then  $q = 1$

$$N^2 = \frac{F M}{B M} \left( \frac{m + \frac{M}{2} (1+1)}{m} \right) \times \frac{895}{h}$$

$$N^2 = \frac{F M}{B M} \left( \frac{m + M}{m} \right) \frac{895}{h}$$

### Problem

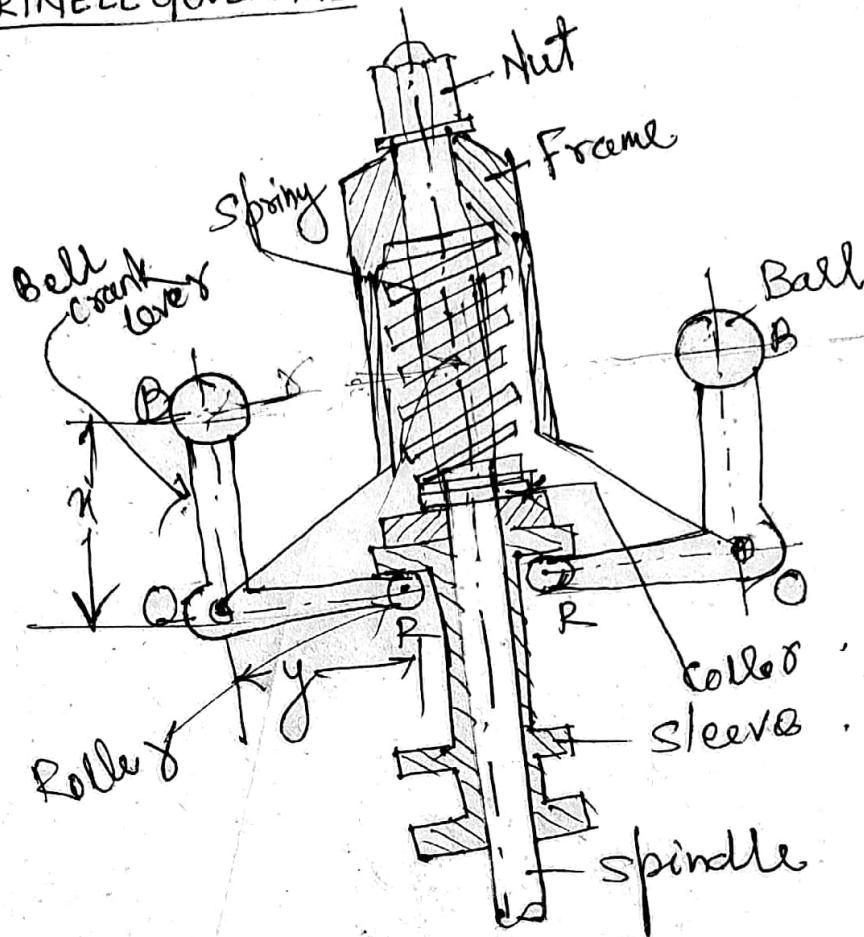
A proell governor has equal arms of length 300 mm. The upper and lower ends of the arm are pivoted on the axis of governor. The extension arms of the lower links are 80 mm long and parallel to the axis of rotation when the radius of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and mass of central load is 100 kg. Determine the range of speed of governor.





# HARTNELL GOVERNOR

(15)



A Hartnell governor is a spring loaded governor. It consists of two bell crank levers pivoted at O and O' to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of vertical arm OB and a roller at the end of horizontal arm OR. A helical spring in compression provides equal downward force on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a Nut up and down on the sleeve.

Let  $m$  = mass of each ball in kg.

$M$  = Mass of sleeve in kg

$r_1$  = Minimum radius of rotation

$r_2$  = Maximum " "

$\omega_1$  = angular speed of governor at <sup>min</sup> radius of rotation

$\omega_2$  = " " " " at max<sup>m</sup> radius of rotation

(10)

$S_1$  = spring force exerted on the sleeve at  $\omega_1$  in N,  
 $S_2$  = " " " " " " at  $\omega_2$  in N,  
 $F_{C1}$  = Centrifugal force at  $\omega_1 = m r_1 \omega_1^2$   
 $F_{C2}$  = " " " " " "  $\omega_2 = m r_2 \omega_2^2$

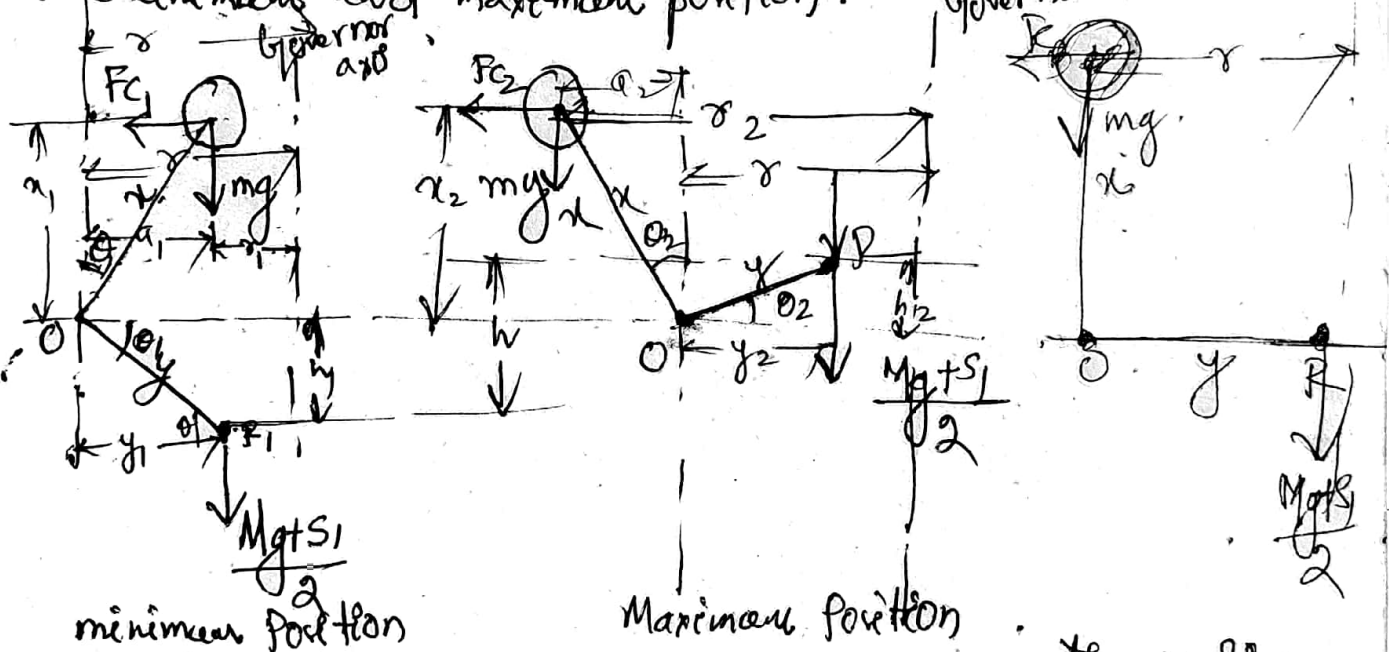
$S$  = stiffness of the spring

$x$  = length of vertical arm

$y$  = length of horizontal or sleeve arm

$\delta$  = distance of fulcrum  $O$  from the governor axis or the radius of rotation when the governor is in mid-position.

Consider the forces acting at one Bell crank lever  
 The minimum and maximum position.



Let  $h$  be the compression of the spring when the radius of rotation changes from  $\delta_1$  to  $\delta_2$

For the minimum position when radius of rotation changes from  $\delta$  to  $\delta_1$ . The compression of the spring or lift of sleeve is  $h_1$

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{\delta - \delta_1}{\delta} \quad \text{--- (1)}$$

Similarly for the maximum position when the radius of rotation changes from  $\delta$  to  $\delta_2$  The compression of spring or lift of sleeve  $h_2$

$$\frac{h_2}{y} = \frac{x_2}{x} = \frac{x_2 - x}{x} \quad \text{--- (2)}$$

(17)

Adding eqn (1) & (2)

$$\frac{h_1 + h_2}{y} = \left( \frac{x - x_1}{x} \right) + \frac{x_2 - x}{x}$$

$$\Rightarrow \frac{h_1 + h_2}{y} = \frac{x_2 - x_1}{x}$$

$$\Rightarrow \frac{h}{y} = \frac{x_2 - x_1}{x}$$

$$\Rightarrow h = \left( \frac{x_2 - x_1}{x} \right) y$$

$$= (x_2 - x_1) \frac{y}{x} \quad \text{--- (3)}$$

Now for minimum position taking moment about O

$$\frac{Mg + S_1}{2} \times y_1 = F_{C1} \times x_1 - m g a_1$$

$$Mg + S_1 = \frac{2}{y_1} (F_{C1} \times x_1 - m g a_1) \quad \text{--- (4)}$$

Again for maximum position taking moment about point O

$$\frac{Mg + S_2}{2} \times y_2 = F_{C2} \times x_2 + m g a_2$$

$$Mg + S_2 = \frac{2}{y_2} (F_{C2} \times x_2 + m g a_2) \quad \text{--- (5)}$$

Subtracting eqn 4 from eqn (5)

$$(Mg + S_2) - (Mg + S_1) = \frac{2}{y_2} (F_{C2} \times x_2 + m g a_2) - \frac{2}{y_1} (F_{C1} \times x_1 - m g a_1)$$

$$\Rightarrow S_2 - S_1 = \frac{2}{y_2} (F_{C2} \times x_2 + m g a_2) - \frac{2}{y_1} (F_{C1} \times x_1 - m g a_1)$$

We know that  $S_2 - S_1 = h \times S$  and  $h = (x_2 - x_1) \frac{y}{x}$

$$\therefore S = \frac{S_2 - S_1}{h} = \frac{(S_2 - S_1) x}{(x_2 - x_1) y} \quad \text{--- (6)}$$

Neglecting the obliquity effect of the arms (i.e.  $x_1 = x_2 = x$  and  $y_1 = y_2 = y$ ) and moment due to weight of balls.

For minimum position

$$\frac{M \cdot g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad Mg + S_1 = 2 F_{C1} \frac{x}{y} \quad \text{--- (7)}$$

For maximum position

$$\frac{Mg + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad Mg + S_2 = 2 F_{C2} \frac{x}{y} \quad \text{--- (8)}$$

Subtracting eqn (7) from 8

$$S_2 - S_1 = 2 F_2 \frac{y}{x} - 2 F_1 \frac{y}{x}$$

$$S_2 - S_1 = 2 \frac{y}{x} (F_2 - F_1)$$

We know that  $S_2 - S_1 = h$   $h = (r_2 - r_1) \frac{y}{x}$

$$S = \frac{S_2 - S_1}{h} = \frac{S_2 - S_1}{(r_2 - r_1) \frac{y}{x}} = \frac{2 \left(\frac{y}{x}\right) (F_2 - F_1)}{(r_2 - r_1) \frac{y}{x}}$$

$$= \frac{2 (F_2 - F_1) \left(\frac{y}{x}\right)^2}{r_2 - r_1}$$

$$S = \frac{2 (F_2 - F_1)}{r_2 - r_1}$$

### Sensitiveness of Governor

Consider two governors running at same speed. When this speed increases or decreases by a certain amount the lift of the governor A is greater than Governor B. It is then said that governor A is more sensitive than Governor B.

→ Sensitiveness may also be defined as the difference between the maximum and minimum equilibrium speed to mean equilibrium speed.

$$\text{Sensitiveness of Governor} = \frac{N_2 - N_1}{N}$$

$$= \frac{N_2 - N_1}{\frac{N_1 + N_2}{2}}$$

$$= \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

where  $N_1$  = minimum equilibrium speed

$N_2$  = Maximum equilibrium speed

$$N = \frac{N_1 + N_2}{2}$$

= mean equilibrium speed



## Stability of Governor

A Governor is said to be stable when for every speed within working range there is a definite configuration. i.e. there is only one radius of rotation at which governor is in equilibrium.

For a stable governor, if the equilibrium speed increases radius of rotation of ball must also increase.

## Isochronous Governor

A governor is said to be isochronous, when the equilibrium speed is constant (Range of speed  $N_2 - N_1 = 0$ ) for all radius of rotation within working range, neglecting friction.

Let us consider Porter Governor

$$N_1^2 = \frac{m + \frac{M}{2}(1+q)}{m} \frac{895}{h_1} \quad \text{--- (1)}$$

$$N_2^2 = \frac{m + \frac{M}{2}(1+q)}{m} \frac{895}{h_2} \quad \text{--- (2)}$$

For isochronism range of speed should be zero

$$N_2 - N_1 = 0 \quad \text{or} \quad N_2 = N_1$$

Therefore eqn 1 = eqn 2

$$\Rightarrow h_1 = h_2$$

which is impossible for a Porter governor. Hence Porter Governor can not be isochronous.

Now consider the case of Hartnell governor running at speed  $N_1$  and  $N_2$  rpm.

$$M \cdot g + S_1 = 2 F_{C1} \times \frac{x}{y} = 2 \times m r_1 \left( \frac{2\pi N_1}{60} \right)^2 \times \frac{x}{y} \quad \text{--- (3)}$$

$$M \cdot g + S_2 = 2 F_{C2} \times \frac{x}{y} = 2 \times m r_2 \left( \frac{2\pi N_2}{60} \right)^2 \times \frac{x}{y}$$

For isochronism  $N_2 = N_1$

$$\frac{M \cdot g + S_1}{M \cdot g + S_2} = \frac{r_1}{r_2}$$

Hence isochronous governor is not in practical use.

## Hunting

A Governor is said to be hunt if the speed of an engine fluctuates continuously above and below mean speed. This is caused by too sensitive governor which changes the fuel supply by a large amount when small change speed of rotation takes place.

### For example

When the load on an engine increases, the engine speed decrease and if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in opening the control valve wide, which will supply the fuel to the engine in excess of its requirements. Due to this engine speed increases rapidly again and governor sleeve rises to the highest position. Due to this movement of sleeve cut off the fuel supply to the engine and thus engine speed falls once again. This cycle is repeated indefinitely.  $\rightarrow$  This cycle fluctuation of speed is called hunting of Governor.

## Effort and Power of a Governor

The effort of a governor is the mean force exerted at the sleeve for given percentage change of speed.

It may be noted that when the governor is running steadily there is no force at the sleeve. But when speed changes there is a resistance at the sleeve which oppose its motion. It is assumed that this resistance which is equal to the effort, varies uniformly from its maximum value to zero while the governor moves from one position to another.

Power of Governor is the workdone at sleeve  
for a given percentage change of speed.

$$\text{Power} = \text{Mean effort} \times \text{lift of sleeve}$$

## Fly Wheel →

①

A flywheel is a device, which store energy during the period when supply energy is more than the requirement and releases the energy when the requirement of energy is more than the supply.

In case of steam engines, IC engines, reciprocating compressors and pumps, the energy developed in one stroke is used for all other stroke.

For ex. In IC engine, the energy is developed only during expansion stroke which is much more than engine load and no energy is developed during suction, compression and exhaust stroke in case of four stroke engine. The excess energy developed during power stroke is absorbed by the flywheel and release it to the crankshaft during other stroke in which no energy is developed.

→ When the flywheel absorb energy, its speed increases and when it releases energy, the speed decreases. Hence it does not maintain a constant speed. It simply reduce the fluctuation of speed

→ A flywheel controls the speed variation caused by fluctuation of the energy (engine turning moment) during each cycle of operation.

on the other hand function of Governor is entirely different from the flywheel. It regulate the mean speed of an engine when there are variation in the load. For ex when load on an engine increases, it becomes necessary to increase the supply of working fluid. On the other hand when the load on engine decreases, less working fluid

# Difference between Flywheel and Governor

## Fly wheel

- ① It is a heavy rotating wheel that reduces the jerk due to variable fluctuation of speed.
- ② It controls the speed variation caused by fluctuation of the engine turning moment during each cycle of operation.
- ③ It runs as long as engine is running.
- ④ It is relatively heavy mechanical device with large moment of inertia.
- ⑤ It stores excess rotational energy to prevent fluctuation of energy.

## Governor

- ① It is a speed control device that controls speed variations caused due to varying load.
- ② It keeps the mean speed of the engine within a specified limit under varying load and by adjusting the fuel supply.
- ③ It runs when the engine does not run at mean speed.
- ④ It is a light mechanical device with relatively less moment of inertia.
- ⑤ It keeps the mean speed of an engine constant by regulating fuel supply.



## Coefficient of Fluctuation of speed

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed.

→ The ratio of maximum fluctuation speed to the mean speed is called the coefficient of fluctuation of speed.

Let  $N_1$  and  $N_2$  = Maximum and minimum speeds in rpm during cycle

$$N = \text{Mean speed in rpm} = \frac{N_1 + N_2}{2}$$

$$\text{Fluctuation of speed} = N_1 - N_2$$

Coefficient of Fluctuation of speed

$$C_s = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}}$$

$$= \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

(in terms of angular speed)

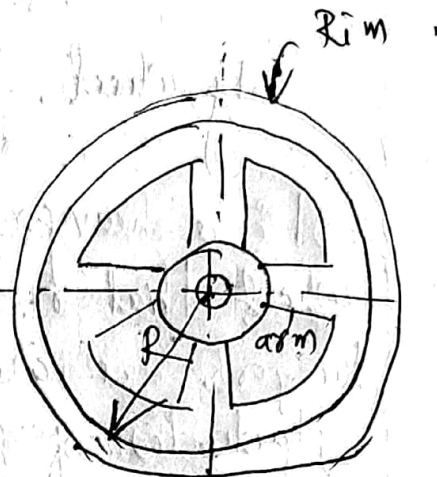
$$= \frac{2(V_1 - V_2)}{V_1 + V_2}$$

in terms of linear speed

## Energy stored in a Flywheel

(3)

A flywheel is shown in fig. We have discussed earlier that when flywheel absorb energy, its speed increases and if release energy, its speed decreases.



Let  $m$  = Mass of flywheel in kg

$k$  = radius of gyration of flywheel in m

$I$  = M.I. of the flywheel about its axis of rotation in  $\text{kg}\cdot\text{m}^2 = mk^2$

$N_1$  and  $N_2$  = Maximum speed and minimum speed during cycle in rpm

$\omega_1$  and  $\omega_2$  = Maximum and minimum angular speed during cycle in rad/sec

Mean speed  $N = \frac{N_1 + N_2}{2}$

Mean angular speed  $\omega = \frac{\omega_1 + \omega_2}{2}$

Coefficient fluctuation speed =  $\frac{N_1 - N_2}{N}$  or  $\frac{\omega_1 - \omega_2}{\omega}$

Mean kinetic energy of the flywheel

$$E = \frac{1}{2} I \omega^2 = \frac{1}{2} mk^2 \omega^2$$

As the speed of flywheel changes from  $\omega_1$  to  $\omega_2$ , the maximum fluctuation of speed energy

$$\begin{aligned} \Delta E &= \text{Maximum K.E} - \text{Minimum K.E} \\ &= \frac{1}{2} I (\omega_1)^2 - \frac{1}{2} I \omega_2^2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2) \end{aligned}$$

$$\Delta E = \frac{1}{2} I (\omega_1 + \omega_2) (\omega_1 - \omega_2)$$

$$= I \frac{(\omega_1 + \omega_2)}{2} (\omega_1 - \omega_2)$$

$$= I \omega (\omega_1 - \omega_2)$$

$$(\because \omega = \frac{\omega_1 + \omega_2}{2})$$

$$= I \omega^2 \left( \frac{\omega_1 - \omega_2}{\omega} \right)$$

( $\because$  multiplying and dividing  $\omega$ )

$$= I \omega^2 C_s = 2 \times \left( \frac{1}{2} I \omega^2 \right) C_s$$

$$= \boxed{m k^2 \omega^2 C_s}$$

$$(I = m k^2)$$

$$= 2 E C_s$$

$$E = \frac{1}{2} I \omega^2$$

$$\boxed{\Delta E = m k^2 \omega^2 C_s} = 2 E C_s$$

The radius of gyration  $k$  may be taken equal to the mean radius of Rim ( $R$ ), because thickness of rim is very small compared to dia of rim.

Hence  $\Delta E = m R^2 \omega^2 C_s = m V^2 C_s$

$$\because (V = R\omega)$$

$V$  = Mean linear velocity

Notes:

Since  $\omega = \frac{2\pi N}{60}$

$$\Delta E = I \omega^2 C_s$$

$$= I \omega^2 \left( \frac{\omega_1 - \omega_2}{\omega} \right)$$

$$= I \omega (\omega_1 - \omega_2)$$

$$= I \times \frac{2\pi N}{60} \left( \frac{2\pi N_1}{60} - \frac{2\pi N_2}{60} \right)$$

$$= \frac{4\pi^2 I N (N_1 - N_2)}{3600}$$

$$= \frac{\pi^2}{900} I N (N_1 - N_2) = \frac{\pi^2}{900} m k^2 N (N_1 - N_2)$$

$$= \frac{\pi^2}{900} m k^2 N^2 \frac{(N_1 - N_2)}{N} = \frac{\pi^2}{900} m k^2 N C_s$$

$$C_s = \frac{N_1 - N_2}{N}$$

$$C_s = \frac{\omega_1 - \omega_2}{\omega}$$

$$\boxed{\Delta E = \frac{\pi^2}{900} m k^2 N C_s}$$

Problem A horizontal cross compound steam engine develops 300 kW at 90 rpm. The coefficient of fluctuation of energy as found from the turning moment is to be 0.1 and the fluctuation of speed is to be kept within  $\pm 0.5\%$  of the mean speed. Find the weight of flywheel required if the radius of gyration is 2 meters. (B)

Sol  $P = 300 \text{ kW} = 300 \times 10^3 \text{ W}$   $N = 90 \text{ rpm}$

$C_E = 0.1$   $k = 2 \text{ m}$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 9.426 \text{ rad/sec}$

Since  $N_1 - N_2 = \pm 0.5\% N$

or  $\omega_1 - \omega_2 = \pm 0.5\% \frac{\omega_1 + \omega_2}{2}$

$\omega_1 - \omega_2 = \pm 1\% \omega$   
 $= 0.01 \omega$

Coefficient of fluctuation of speed

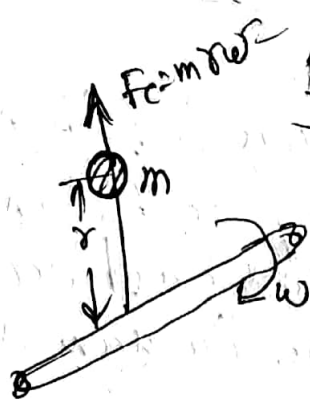
$C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.01$

Workdone per cycle  $= P \times \frac{60}{N} = 300 \times 10^3 \times \frac{60}{90}$   
 $= 200 \times 10^3 \text{ N}\cdot\text{m}$

Maximum fluctuation of energy  $\Delta E = \text{workdone/cycle} \times C_E$   
 $= 200 \times 10^3 \times 0.01$   
 $= 20 \times 10^3 \text{ N}\cdot\text{m}$

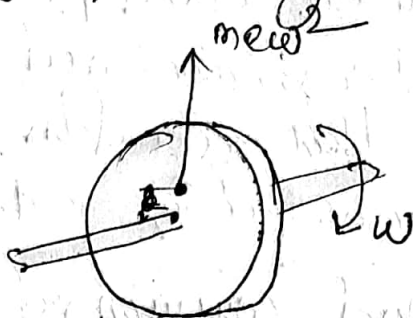
Maximum fluctuation of energy ( $\Delta E$ )

$20 \times 10^3 = m k^2 \omega^2 = m \times 2^2 \times (9.426)^2 \times 0.01$   
 $m = 5630 \text{ kg}$



## Balancing of Rotating Masses

①



Sometimes the unbalance force is produced in rotary or reciprocating machinery due to inertia forces associated with the moving masses.

→ Balancing is the process of designing or modifying machinery so that unbalance is reduced or eliminated entirely.

→ When a mass is moving in a circular path, it experiences a centripetal acc<sup>n</sup> and a force is required to produce it. An equal and opposite force acting radially outward direction is known as Centrifugal force. This is the disturbing force on the axis of rotation. The magnitude of centrifugal force is constant, but the direction changes with the rotation of masses.

→ In a revolving rotor, the centrifugal force remains balanced when the center of mass is on the axis of rotation of the shaft.

→ When the center of mass (rotor) does not lie on the axis or there is an eccentricity, an unbalanced force is produced (Fig b) ( $m\omega^2 r$ ). This type of unbalance is very common. For ex  
① steam turbine, engine crankshaft, rotary compressor and centrifugal pump.



→ Most of the serious problem encountered in high speed machinery are direct result of unbalanced forces. due to this vibration of machine part occurs, it occurs noise and there are human discomfort.

→ The most common approach to balancing is by addition or removal of mass from various machine members.

There are two basic type of Balancing (cond)

① Balancing of Rotating masses

② Balancing of Reciprocating masses

static Balancing: - A system of rotating masses is said to be static balancing if the combined mass center of the system lies on the axis of rotation.

Balancing of Rotating Masses

Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibration in it.

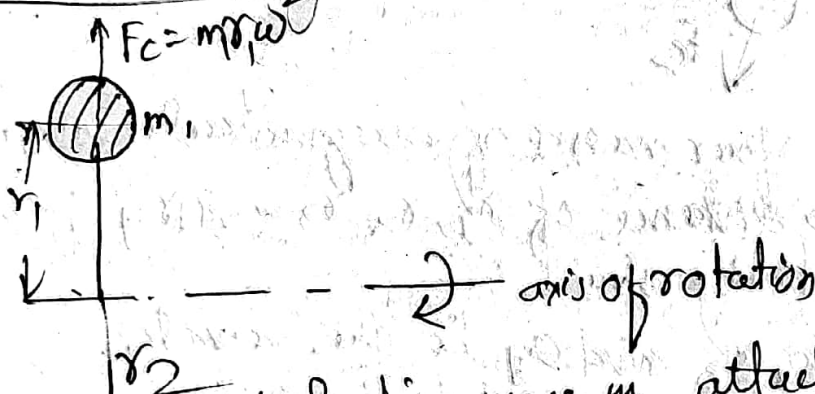
→ In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft. This is done in such a way that the centrifugal force of both masses are equal and opposite.

→ This process of providing the and mass in order to counteract the effect of centrifugal force of 1st mass is called Balancing of Rotating masses.

The following cases are important

1. Balancing of single rotating mass by a single mass rotating in the same plane.
2. Balancing of different masses rotating in the same plane.
3. Balancing of different masses rotating in different plane.

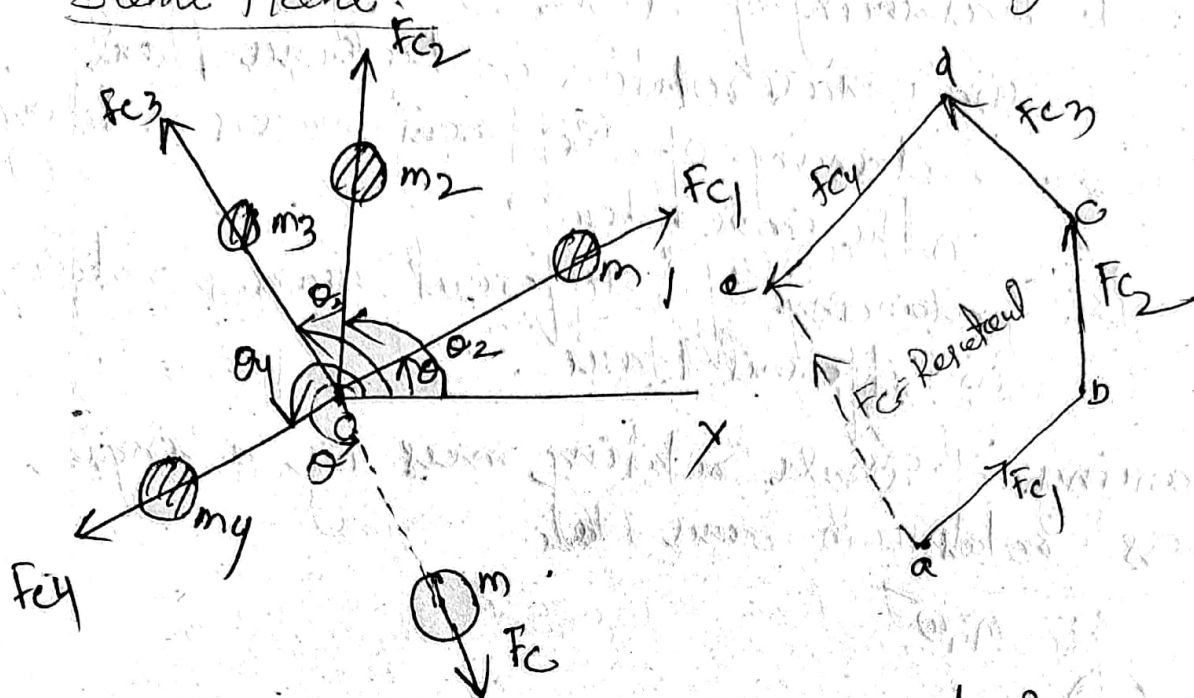
1. Balancing of single rotating mass by a single mass rotating in same plane



Consider a disturbing mass  $m_1$  attached to a shaft rotating at  $\omega$  rad/sec.  
 Let  $r_1$  be distance of mass from axis of rotation.  
 Centrifugal force by the mass  $F_{c1} = m_1 r_1 \omega^2$  — (1)  
 This centrifugal force is radially outward directed.  
 This produces bending moment of this shaft.  
 In order to counteract the effect of this force a balancing mass ( $m_2$ ) is attached in the same plane of rotation as that of disturbing mass ( $m_1$ ) such that the centrifugal force of two masses are equal and opposite.

Centrifugal force of mass ( $m_2$ )  $F_{c2} = m_2 r_2 \omega^2$  — (2)  
 $m_1 r_1 \omega^2 = m_2 r_2 \omega^2 \Rightarrow m_1 r_1 = m_2 r_2$  (Balanced)

# Balancing of Several Masses Rotating in the Same Plane.



→ Consider a four masses of magnitude,  $m_1, m_2, m_3$  and  $m_4$  at a distances of  $r_1, r_2, r_3$  and  $r_4$  from the axis of rotating shaft.

→ Let  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  be the angles of these masses with the horizontal line OX.

→ Let all the masses rotate about an axis through O perpendicular to the plane of paper with a constant angular speed  $\omega$  rad/sec.

The magnitude of Balancing mass may be found out by two methods.

- ① Analytical Method
- ② Graphical Method

## 1. Analytical Method

First of find out the Balancing of mass  
Centrifugal force exerted by each mass such  
a)  $F_{c1} = m_1 r_1 \omega^2$   $F_{c2} = m_2 r_2 \omega^2$   $F_{c3} = m_3 r_3 \omega^2$   $F_{c4} = m_4 r_4 \omega^2$   
∴ constant for all masses Hence  $F_{c1} = m_1 r_1$  ,  $F_{c2} = m_2 r_2$

(ii) Resolve the centrifugal forces horizontally and vertically and find their sum.  
 Sum Horizontal component of centrifugal force

$$\Sigma H = m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 \dots$$

Vertical component of centrifugal force

$$\Sigma V = m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + \dots$$

(iii) Magnitude of centrifugal force

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

(iv) If  $\theta$  is the angle, which the resultant force makes with horizontal

$$\tan \theta = \frac{\Sigma V}{\Sigma H}$$

(v) The balancing force is then equal to the resultant force, but in opposite direction.

(vi) Now find out magnitude of balancing mass

$$F_c = m r$$

where  $m$  = Balancing Mass  
 $r$  = radius of rotation

### Graphical Method

(i) First of all draw the space diagram with the positions of several masses as shown in fig.

(ii) Find out centrifugal force (or product of mass and radius of rotation) of each mass.

(iii) Now draw the vector diagram with obtained centrifugal forces (or product of the masses and their radius of rotation) as represents centrifugal force of  $m_1, (m_1 r_1)$  in magnitude and direction to some suitable scale.



Similarly draw  $bc$ ,  $cd$  and  $de$  represents centrifugal force of masses  $m_2$ ,  $m_3$  and  $m_4$  (or,  $m_2 r_2$ ,  $m_3 r_3$ ,  $m_4 r_4$ )

(iv) Now as per the polygon law of forces, the closing side  $a e$  represents the resultant force in magnitude and direction.

(v) Balancing force is equal to the resultant force but in opposite direction.

(vi) Now find out the balancing mass  $m'$  and radius of rotation ( $r$ )

$$m r \omega^2 = \text{Resultant centrifugal force}$$

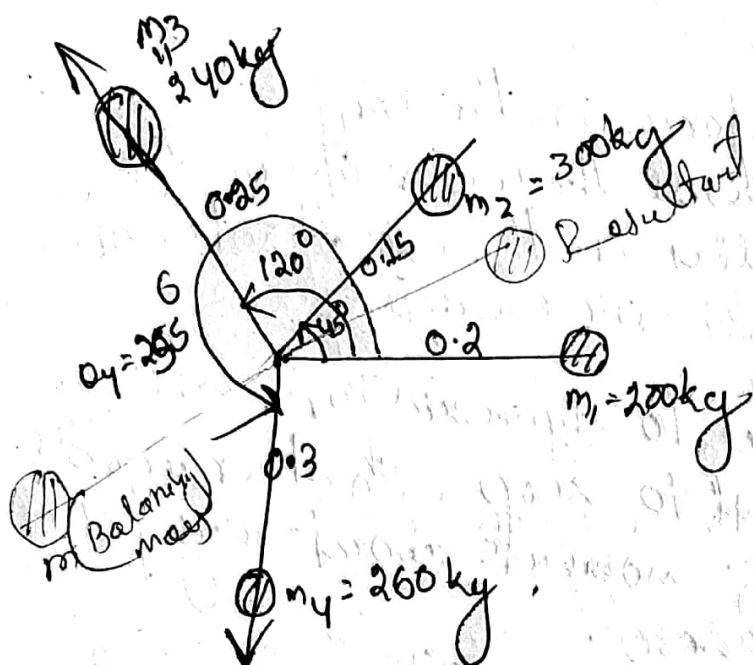
$$m r = \text{resultant of } m_1 r_1, m_2 r_2, m_3 r_3 \text{ and } m_4 r_4$$

Example Four masses  $m_1$ ,  $m_2$ ,  $m_3$  and  $m_4$  are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angle bet<sup>n</sup> two successive masses are  $45^\circ$ ,  $75^\circ$  and  $135^\circ$ . Find the position and magnitude of balance mass required if the radius of rotation is 0.2 m.

Data Given

$m_1 = 200 \text{ kg}$	$m_2 = 300 \text{ kg}$	$m_3 = 240 \text{ kg}$
$m_4 = 260 \text{ kg}$	$r_1 = 0.2 \text{ m}$	$r_2 = 0.15 \text{ m}$
$r_4 = 0.3 \text{ m}$	$r_3 = 0.25 \text{ m}$	
$\theta_1 = 0^\circ$	$\theta_2 = 45^\circ$	$\theta_3 = 45^\circ + 75^\circ = 120^\circ$
$\theta_4 = 45^\circ + 75^\circ + 135^\circ = 255^\circ$		$r = 0.2 \text{ m}$





### Analytical Method

$$\begin{aligned}\Sigma H &= m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 + m_4 r_4 \cos \theta_4 \\ &= 200 \times 0.2 \cos 0^\circ + (300 \times 0.15) \cos 45^\circ + 240 \times 0.25 \cos 120^\circ \\ &\quad + (260 \times 0.3) \cos 255^\circ \\ &= 21.6 \text{ kg.m.}\end{aligned}$$

Resolving vertically.

$$\begin{aligned}\Sigma V &= m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3 + m_4 r_4 \sin \theta_4 \\ &= (200 \times 0.2) \sin 0^\circ + (300 \times 0.15) \sin 45^\circ + 240 \times 0.25 \sin 120^\circ \\ &\quad + (260 \times 0.3) \sin 255^\circ \\ &= 8.5 \text{ kg.m.}\end{aligned}$$

$$\therefore \text{or Resultant } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2 \text{ kg.m}$$

$$F_c = R$$

$$\Rightarrow m \cdot r = 23.2 \text{ kg.m}$$

$$\Rightarrow m = \frac{23.2}{0.2} = 116 \text{ kg}$$

$$\tan \theta' = \frac{\Sigma V}{\Sigma H} = \frac{8.5}{21.6}$$

$$\theta' = 21.48^\circ$$

Since  $\theta'$  is the angle of resultant  $R$  from horizontal

But the angle of Balancing mass from horizontal  
 $= 180^\circ + \theta'$   
 $= 180^\circ + 21.48^\circ$   
 $= 201.48^\circ$

# Dynamic Balancing

②

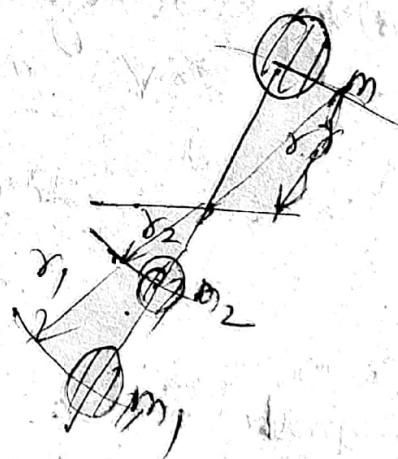
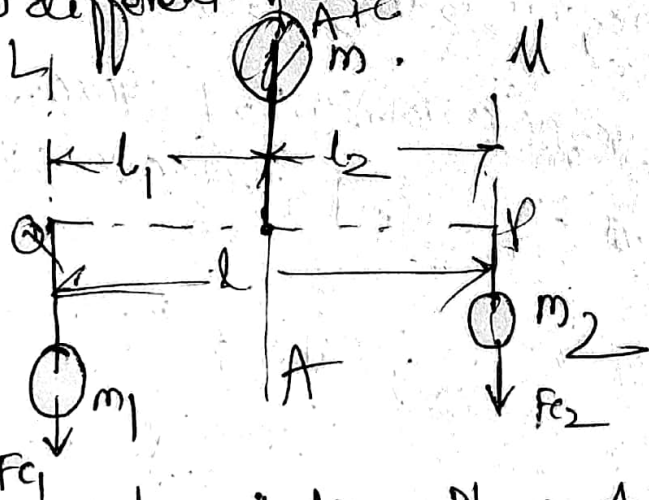
① The net dynamic force on the shaft is equal to zero. In other words the center of masses of the system must lie on the axis of rotation. This is the cond for static Balancing.

2. The net couple due to dynamic force acting on the shaft is equal to zero. In other words the algebraic sum of moments about any point in the plane must be zero.

The cond ① & ② together gives dynamic Balancing

## Balancing of Rotating masses in different Plane

Consider a disturbing mass  $m$  lying in the plane A to be balanced by two rotating masses  $m_1$  and  $m_2$  lying in two different planes.



$l_1$  = distance between Plane A & L  
 $l_2$  = " " " M & A  
 $L$  = " " " L & M

Centrifugal force exerted by the mass  $m$  in plane  $A$

$$F_c = m r \omega^2$$

Centrifugal force by  $m_1$  in Plane  $L = F_{c1} = m_1 r_1 \omega^2$

" " "  $m_2$  in Plane  $M = F_{c2} = m_2 r_2 \omega^2$

Net force acting on the shaft  $= 0$

$$F_c = F_{c1} + F_{c2}$$

$$\Rightarrow m r \omega^2 = m_1 r_1 \omega^2 + m_2 r_2 \omega^2$$

$$\Rightarrow m r = m_1 r_1 + m_2 r_2 \quad \text{--- (i)}$$

In order to find out the magnitude of balancing force in the Plane  $L$  (or the dynamic force at the Q of bearing shaft). Take moment about  $P$

$$F_{c1} \times L = F_{c2} \times L_2 \quad \text{or} \quad m_1 r_1 \omega^2 L = m r \omega^2 L_2$$

$$\Rightarrow m_1 r_1 L = m r L_2 \Rightarrow \boxed{m_1 r_1 = m r \frac{L_2}{L}} \quad \text{--- (ii)}$$

Similarly, in order to find out the balancing force in Plane  $M$ , take moment about  $Q$  which is the point of intersection of Plane  $L$  and axis of rotation

$$F_{c2} \times l = F_c \times l_1$$

$$\Rightarrow m_2 r_2 \omega^2 \times L = m r \omega^2 L_1$$

$$\Rightarrow \boxed{m_2 r_2 = m r \frac{L_1}{L}} \quad \text{--- (iii)}$$

eqn (i) is static balancing. But in order to achieve dynamic balance, eqn (ii) and (iii) must also be satisfied.

← RP →

## (10) Balancing of Several Masses Rotating in different Planes

In order to balance the several revolving masses in different plane the following two cond<sup>n</sup> must be satisfied.

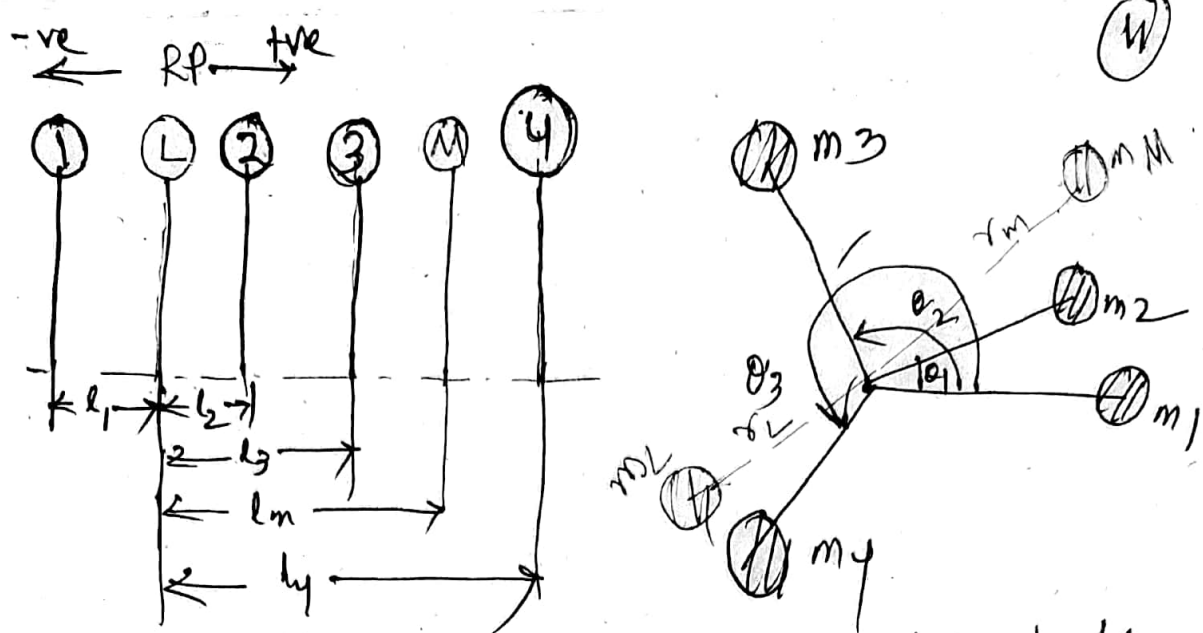
1. The force in the reference plane must be balanced.
2. The couple about the reference plane must be balanced.

When the several masses revolve in different planes may be transferred to a Reference plane (R.P) which may be defined as the plane passing through axis of rotation and perpendicular to it.

→ The centrifugal forces of different plane and couples due to centrifugal forces of different planes may be transferred to the R.P.

→ Now consider four masses  $m_1, m_2, m_3$  and  $m_4$  revolving in the planes 1, 2, 3 and 4 respectively as shown in fig.

The angular position of the masses are shown in fig. The magnitude of balancing masses  $m_L$  and  $m_H$  in planes L and H may be obtained as



The magnitude of balancing masses  $m_L$  and  $m_M$  in planes 1 and M may be obtained as discussed below.

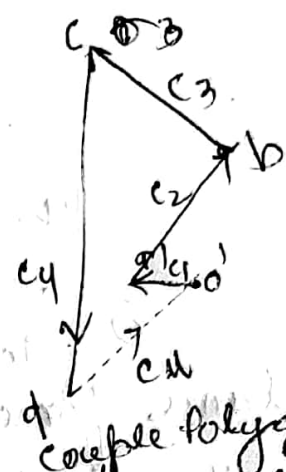
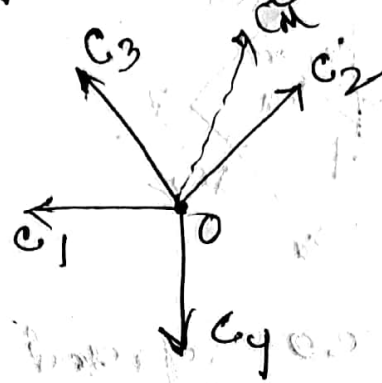
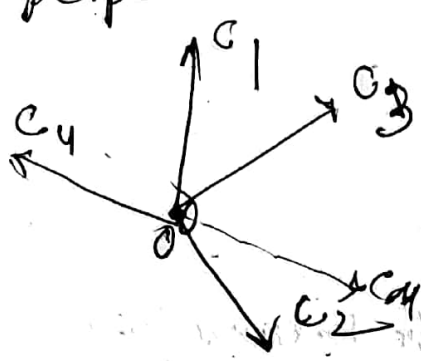
1. Take one of the reference plane L (R.P). the distance of all other plane to the left of R.P is -ve and those to right is +ve.
2. Tabulate the data.

Plane	Mass $m$	Radius $r$	Centrifugal force $m\omega^2 r$	Distance from plane L	Couple $m\omega^2 rL$
1	$m_1$	$r_1$	$m_1 r_1$	$-L_1$	$-m_1 r_1 L_1$
L	$m_L$	$r_L$	$m_L r_L$	0	0
2	$m_2$	$r_2$	$m_2 r_2$	$L_2$	$m_2 r_2 L_2$
3	$m_3$	$r_3$	$m_3 r_3$	$L_3$	$m_3 r_3 L_3$
M	$m_M$	$r_M$	$m_M r_M$	$L_M$	$m_M r_M L_M$
4	$m_4$	$r_4$	$m_4 r_4$	$L_4$	$m_4 r_4 L_4$

3. A couple may be represented by a vector drawn perpendicular to the plane of couple. The couple  $C_1$  introduced by transferring  $m_1$  to the reference plane L is proportional to  $m_1 r_1 L_1$  and act in a plane through  $O_{m_1}$ .



and perpendicular to the paper. The vector representing this couple is drawn on the plane of the paper and perpendicular to  $OM_1$ , as shown by  $OC_1$ .



Similarly  $OC_2$ ,  $OC_3$  and  $OC_4$  are drawn perpendicular to  $OM_2$ ,  $OM_3$  and  $OM_4$  respectively.

The couple vectors as discussed above turned to counter clockwise through a right angle for convenience of drawing we see that their relative position remains unaffected. Now the vector  $OC_2$ ,  $OC_3$ ,  $OC_4$  are parallel and in the same direction as  $OM_2$ ,  $OM_3$  and  $OM_4$ , while the vector  $OC_1$  is parallel to  $OM_1$  but in opposite direction.

Hence the couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inwards for the mass on the other side of the reference plane.

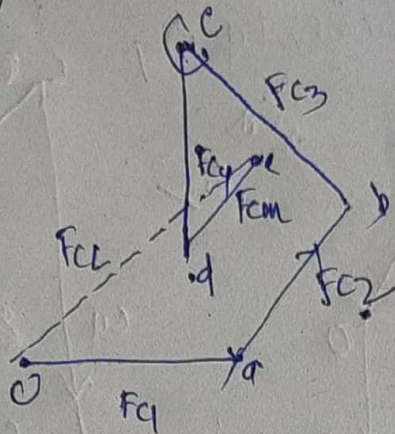
Now draw the couple polygon. The vector  $d'O'$  represents balanced couple. Some balanced couple  $C_m$  is proportional  $m_m \delta_m L_m$

$$C_m = m_m \delta_m L_m = \text{vector } d'O'$$

$$m_m = \frac{\text{vector } d'O'}{\delta_m L_m}$$



Now draw force polygon



The balance force  $e_o$  represent the balance force  
since it is proportional to  $m \cdot r$

$$m_L r_L = \text{vector } e_o$$

$$m_L = \frac{\text{vector } e_o}{r}$$

Problem A shaft carries four masses A, B, C, D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radius 80 mm, 100 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angle between the crank measured anticlockwise are A to B  $45^\circ$ , B to C  $70^\circ$  and C to D  $120^\circ$ . The balancing masses are to be placed in plane X and Y. The distance between the plane A and X is 100 mm between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find the magnitude and angular position.



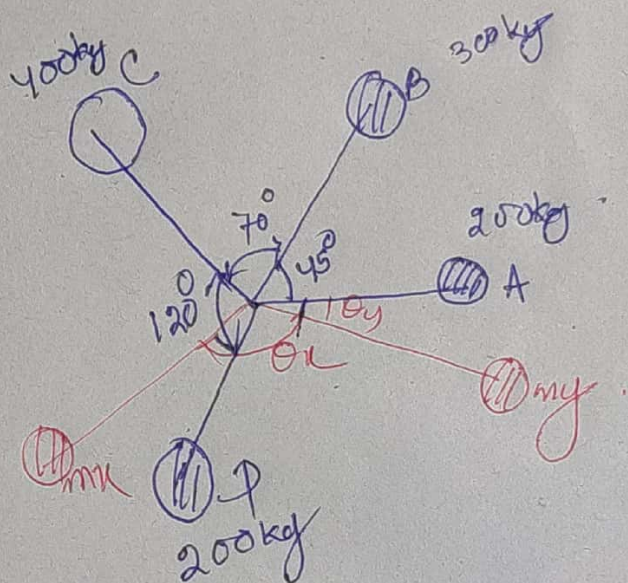
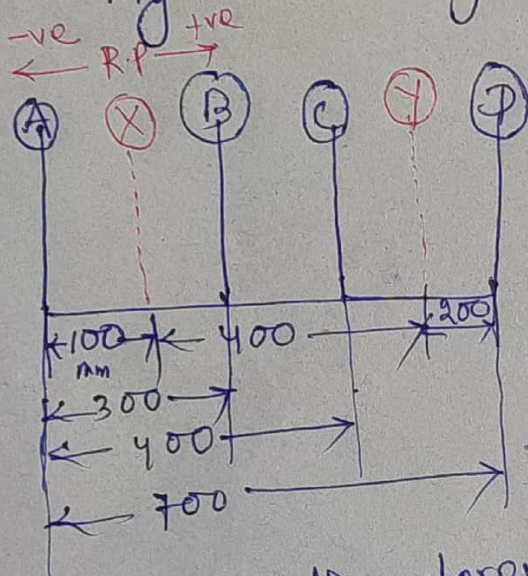
# Data Given

$$m_A = 200 \text{ kg}, m_B = 300 \text{ kg}, m_C = 400 \text{ kg}, m_D = 200 \text{ kg}$$

$$r_A = 80 \text{ mm} = 0.08 \text{ m}, r_B = 70 \text{ mm} = 0.07 \text{ m}, r_C = 60 \text{ mm} = 0.06 \text{ m}, r_D = 80 \text{ mm} = 0.08 \text{ m}$$

$$r_x = r_y = 100 \text{ mm} = 0.1 \text{ m}$$

$m_x$  = Balancing mass to be placed in plane x  
 $m_y$  = Balancing mass to be placed in plane y

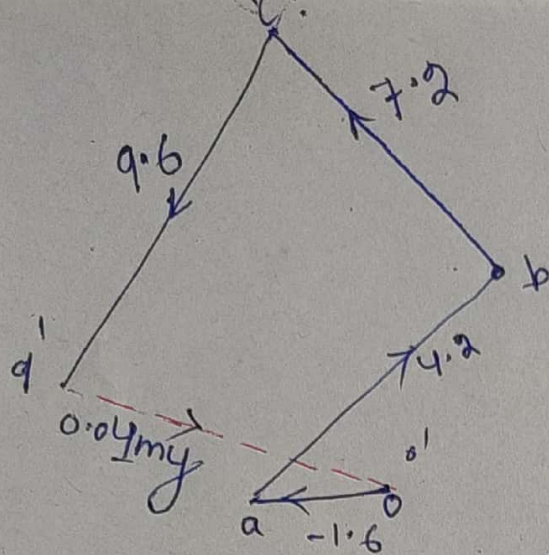


Assume X is the reference plane

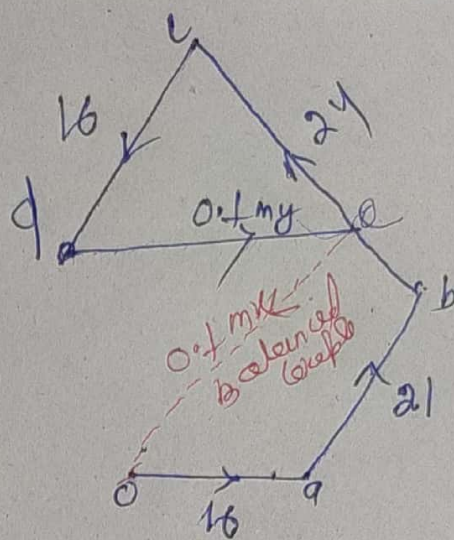
Plane	Mass kg	Radius (r) in m	Centrifugal force $\omega^2 r$ $= (mr) \text{ kg m}$	distance from Plane X = L meter	Couple $\div \omega^2$ $= m r L \text{ kg m}^2$
A	200 kg	0.08	$200 \times 0.08 = 16$	-0.1	$-16 \times 0.1 = -1.6$
X (R.P.)	$m_x$	0.1	$0.1 m_x$	0	0
B	300	0.07	$300 \times 0.07 = 21$	0.2	$21 \times 0.2 = 4.2$
C	400	0.06	$400 \times 0.06 = 24$	0.3	$24 \times 0.3 = 7.2$
Y	$m_y$	$r_y = 0.1$	$0.1 m_y$	0.4	$0.1 m_y \times 0.4 = 0.04 m_y$
D	200	0.08	$200 \times 0.08 = 16$	0.6	$16 \times 0.6 = 9.6$

100 mm m\_y





Couple Polygon.



15

1. If of all, draw the couple polygon from table to some suitable scale. The vector  $d'o$  represents the balanced couple, since the balanced couple is proportional to  $0.04 m_y$

$$0.04 m_y = \text{vector } d'o = 7.3 \text{ kgm}^2$$

$$\Rightarrow m_y = 182.5 \text{ kg}$$

2. The angular position of mass  $m_y$  is obtained by drawing  $o m_y$  parallel to vector  $d'o$ . By measurement the angular position of  $m_y$  is  $\theta_y = 12^\circ$  in the clockwise direction of  $m_x$ .

3. Now draw the force polygon from data ( $m_x$ ) from table. The vector  $eo$  represents balanced force. Since the balanced force is proportional to  $0.1 m_x$

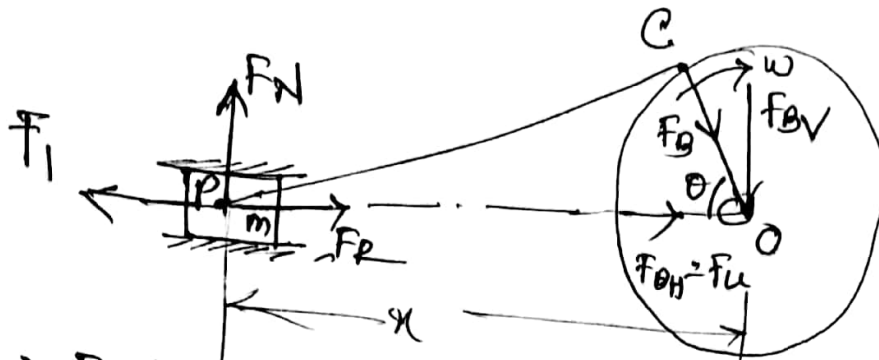
$$0.1 m_x = \text{vector } eo = 35.5 \text{ kgm}$$

$$m_x = 355 \text{ kg}$$

The angular position of the  $m_x$  is obtained by drawing  $o m_x$  to vector  $eo$ . By measurement the angular position of  $m_x$  is  $\theta_x = 145^\circ$  in the clockwise from  $m_y$ .



# Balancing of Reciprocating Masses



$F_R$  = Force Required to accelerate the reciprocating parts

$F_I$  = Inertia force due to reciprocating parts.

$F_N$  = Normal force acting on the cross-head guides.

$F_B$  = Force acting on the crankshaft bearing or main bearing

The resultant of all forces due to inertial forces only is known as unbalance force. Thus if the resultant of all the forces due to inertial effect is zero, then there will be no inertial unbalance force, but even then an unbalanced couple or shaking couple will be present.

→ Since  $F_R = F_I$  but opposite direction they balance each other.

→ The horizontal component of  $F_B$  (i.e.  $F_{BH}$ ) acting along reciprocation is also equal and opposite to  $F$ . This force  $F_{BH} = F_u$  is an unbalance force and required to be balanced.



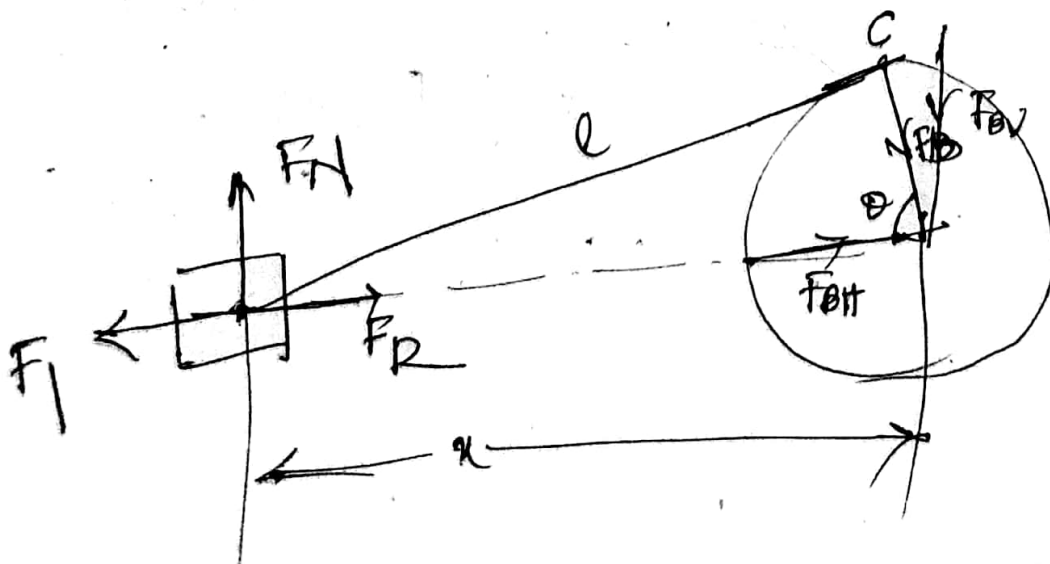
→ The force  $F_H$  and the vertical component of  $F_B$  (ie  $F_{BV}$ ) are equal and opposite and thus form a shaking couple of magnitude  $(F_H \times x$  or  $F_{BV} \times x$

→ From the above we see that effect of reciprocating parts is to produce a unbalance force and a unbalance couple. They cause vibration of parts.

→ Thus purpose of balancing the reciprocating masses is to eliminate unbalance force and couple. This can be done by adding a appropriate balancing mass, but it is usually not practical to eliminate them completely. Hence reciprocating masses are partially balanced.

## Primary and Secondary Unbalanced Forces of Reciprocating masses

Consider a reciprocating engine Mechanism



Let

$m$  = Mass of Reciprocating parts.

$l$  = length of connecting rod (cr)

$r$  = Radius of crank (oc)

$\theta$  = Angle of inclination of crank with line of stroke  $PO$ .

$\omega$  = angular speed of crank.

$n$  = Ratio of length of connecting rod to the radius =  $l/r$

acc of Reciprocating parts

$$a_R = \omega^2 r \left( \cos \theta + \frac{\cos 2\theta}{n} \right)$$

Inertia force due to reciprocating parts.

$$F_I = F_R = \text{mass} \times \text{acc} = m \omega^2 r \left( \cos \theta + \frac{\cos 2\theta}{n} \right)$$

Since  $F_{OH} = F_{OI}$  unbalanced force and denoted by  $F_u$

$$\begin{aligned} \text{Unbalanced force } F_u &= m \omega^2 r \left( \cos \theta + \frac{\cos 2\theta}{n} \right) \\ &= m r \omega^2 \cos \theta + m r \omega^2 \frac{\cos 2\theta}{n} \\ &= F_P + F_S \end{aligned}$$

The expression  $m r \omega^2 \cos \theta = F_P$  that is primary unbalanced force

The expression  $m r \omega^2 \frac{\cos 2\theta}{n} = F_S$  is called secondary unbalanced force.

Note-I Primary unbalanced force  $F_p = m r \omega^2 \cos \theta$   
 it is maximum when  $\theta = 0^\circ$  or  $180^\circ$ , Thus  
 primary unbalanced force is max<sup>m</sup> twice in one revolution  
 of the crank.  
 Max<sup>m</sup> primary unbalanced force  
 $F_{p \max} = m r \omega^2$

Note-II Secondary unbalanced force  $F_s = m r \omega^2 \frac{\cos 2\theta}{n}$   
 $F_s$  is max<sup>m</sup> when  $\theta = 0^\circ, 90^\circ, 180^\circ$  and  $360^\circ$   
 Hence secondary unbalance force is max<sup>m</sup> four times  
 in one revolution of crank

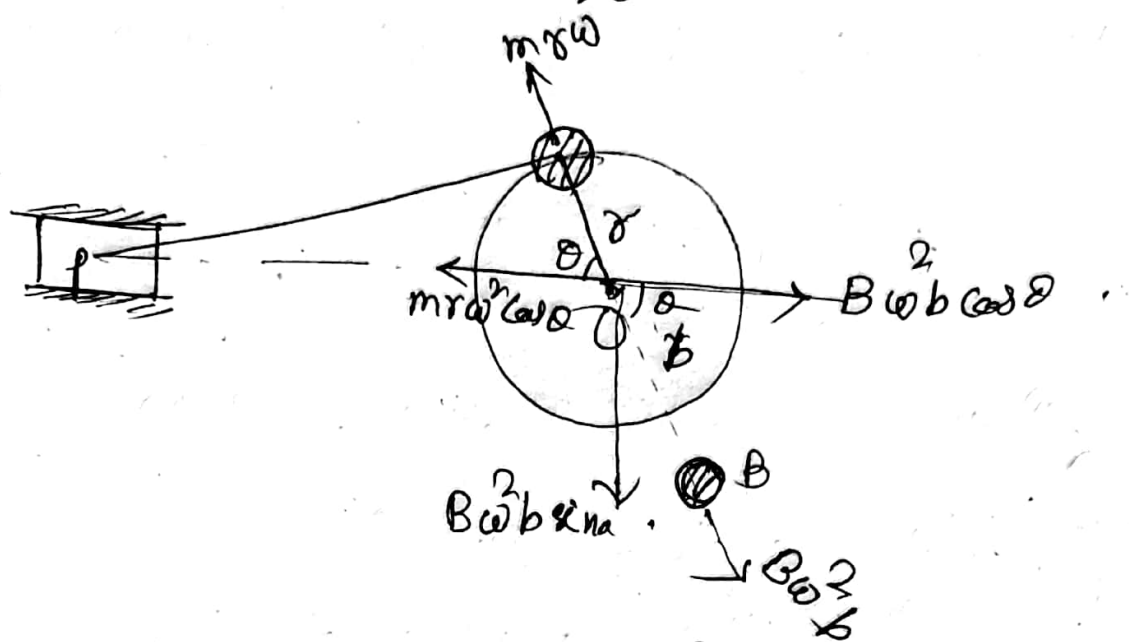
Max<sup>m</sup> secondary unbalance force  
 $F_{s \max} = \frac{m r \omega^2}{n}$

Note-III From above it is seen that secondary  
 unbalance force is  $\frac{1}{n}$  times the max<sup>m</sup> primary  
 unbalance force.

Note-IV In case of moderate speed secondary  
 unbalanced force is small hence neglected

Note-V The unbalance force due to reciprocating  
 parts varies in magnitude but constant  
 in direction which due to revolving mass  
 the unbalance force is constant in magnitude  
 & varies in direction.

# Partial Balancing of Primary Force in a Reciprocating Engine



The primary unbalanced force ( $m\omega^2 r \cos \theta$ ) may be considered as component of centrifugal force produced by rotating mass  $m$  placed at the crank radius  $r$ .

→ The primary force act from O to P along the line of stroke, Hence Balancing of primary force is considered as equivalent Balancing of mass  $m$  rotating at the crank radius  $r$ .

This is balanced by having a mass  $B$  at a radius  $b$  placed diametrically opposite to the crank pin C. Centrifugal force due to mass  $B = B\omega^2 b$   
Horizontal component of this force =  $Bb\omega^2 \cos \theta$

The primary force is balanced if

$$B\omega^2 b \cos \theta = m\omega^2 r \cos \theta$$

$$B \cdot b = m \cdot r$$

It shows that the primary unbalanced force is balanced if  $Bb = mr$ .

but the centrifugal force produced due to revolving mass B. has also vertical component i.e.  $Bb\omega^2 \sin\theta$  this force remains unbalanced.

The maximum value of this force is when  $\theta = 90^\circ$  and  $270^\circ$  which is same as the max<sup>m</sup> value of primary force i.e.  $m r \omega^2$ .

From the above discussion we see that in the 1st case, the primary unbalanced force act along the line of stroke, where as in 2nd case the unbalanced force act along the perpendicular to the line of stroke. The maximum value of the force remains same both the cases. It is thus obvious that the effect of the above method of balancing is to change the direction of the maximum unbalanced force from the line of stroke. As a compromise let a fraction  $c$  of the reciprocating masses is balanced such that

$$C m r = B \cdot b$$

$$\begin{aligned} \therefore \text{Unbalanced force along the line of stroke} &= m r \omega^2 \cos\theta - B \omega^2 \cdot b \cos\theta \\ &= m r \omega^2 \cos\theta - C m r \omega^2 \cos\theta \\ &= (1 - C) m r \omega^2 \cos\theta. \end{aligned}$$

$$\text{Unbalanced force along the perpendicular to the line of stroke} = B \omega^2 b \sin\theta = C m \omega^2 r \sin\theta.$$



(2)

Resultant of unbalanced force at any inst.

$$= \sqrt{((1-c)m\omega^2 r \cos\theta)^2 + (cm\omega^2 r \sin\theta)^2}$$

$$= m\omega^2 r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

Note: If the balancing mass is required to balance the revolving masses as well as reciprocating masses then.

$$B.b = m_1 r + cmr$$

$$= (m_1 + cm) r$$

$m_1$  = Magnitude of Revolving masses  
 $m_2$  = Magnitude of Reciprocating masses

### Problem 2

A single cylinder reciprocating engine has speed 2400 rpm, stroke 300 mm, mass of reciprocating parts at 150 mm radius 37 kg of 100-300 of reciprocating parts and all revolving parts to be balanced. Find

1. The balance mass required at a radius of 400 mm
2. Residual unbalance force when the crank was rotated  $60^\circ$  from inner dead center.

Given

$$N = 240 \text{ rpm}, \quad \omega = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/sec}$$

$$\text{Stroke} = 300 \text{ mm} = 0.3 \text{ m} \quad m = 50 \text{ kg}, \quad m_1 = 37 \text{ kg}$$

$$r = 150 \text{ mm} = 0.15 \text{ m} \quad c = \frac{1}{3}$$

1. Balance mass required

let  $B$  = Balance mass required  
 $b$  = radius of rotation of balance mass  
 $m_{\text{eff}} = 400 \text{ mm} = 0.4 \text{ m}$

$$B \cdot b = m_1 r + c m r$$

$$\Rightarrow B \times 0.4 = (m_1 + c m) r$$

$$\Rightarrow B \times 0.4 = \left( 37 + \frac{1}{3} \times 50 \right) 0.15 = 10.55$$

$$B = 26.38 \text{ kg}$$

2. Residual unbalanced force

let  $\theta$  = Crank angle from inner dead center =  $60^\circ$

Residual unbalanced force

$$= m \omega^2 \sqrt{(1-c)^2 \cos^2 \theta + c^2 \sin^2 \theta}$$

$$= 50 (25.14)^2 \times 0.15 \sqrt{\left(1 - \frac{1}{3}\right)^2 \cos^2 60^\circ + \left(\frac{1}{3}\right)^2 \sin^2 60^\circ}$$

$$= 2899 \text{ N}$$

## Introduction

The study of vibrations is concerned with the oscillatory motion of bodies and the forces associated with them. All bodies possessing mass and elasticity are capable of vibrating. Thus most engineering machines and structures experiences vibration to some degree and their design generally requires consideration of their oscillatory behaviour. The oscillatory motion of the system may be objectionable or necessary for performing a task.

The objective of the designer is to control the vibration when it is objectionable and to enhance the vibration when it is useful. Objectionable or undesirable vibration in machine may cause the loosening of parts, its malfunctioning or its failure. The useful vibration helps in the design of shaker in foundries, vibrators in testing machines etc. Sometimes vibrations are bad and other times they are good.

### 1.1. Causes of vibration: - The main causes of vibration are:-

- 1) **Unbalanced forces in the machine.** These forces are produced from within the machine itself because of non-uniform material distribution in a rotating machine element.
- 2) **Dry friction between the two mating surfaces:** This is what known as self-excited vibration.
- 3) **External excitations.** These excitations may be periodic, random or of the nature of an impact produced external to the vibrating system.
- 4) **Elastic nature of the system**
- 5) **Earth quakes.** These are responsible for the failure of many buildings, dams etc.
- 6) **Winds.** These may cause the vibrations of transmission and telephone lines under certain conditions.

The effect of vibrations is excessive stresses, undesirable noise, looseness of parts and partial or complete failure of parts. Inspite of these harmful effects the vibration phenomenon does have, some uses also e.g. in musical instruments, vibrating screens, shakers, stress relieving.etc.

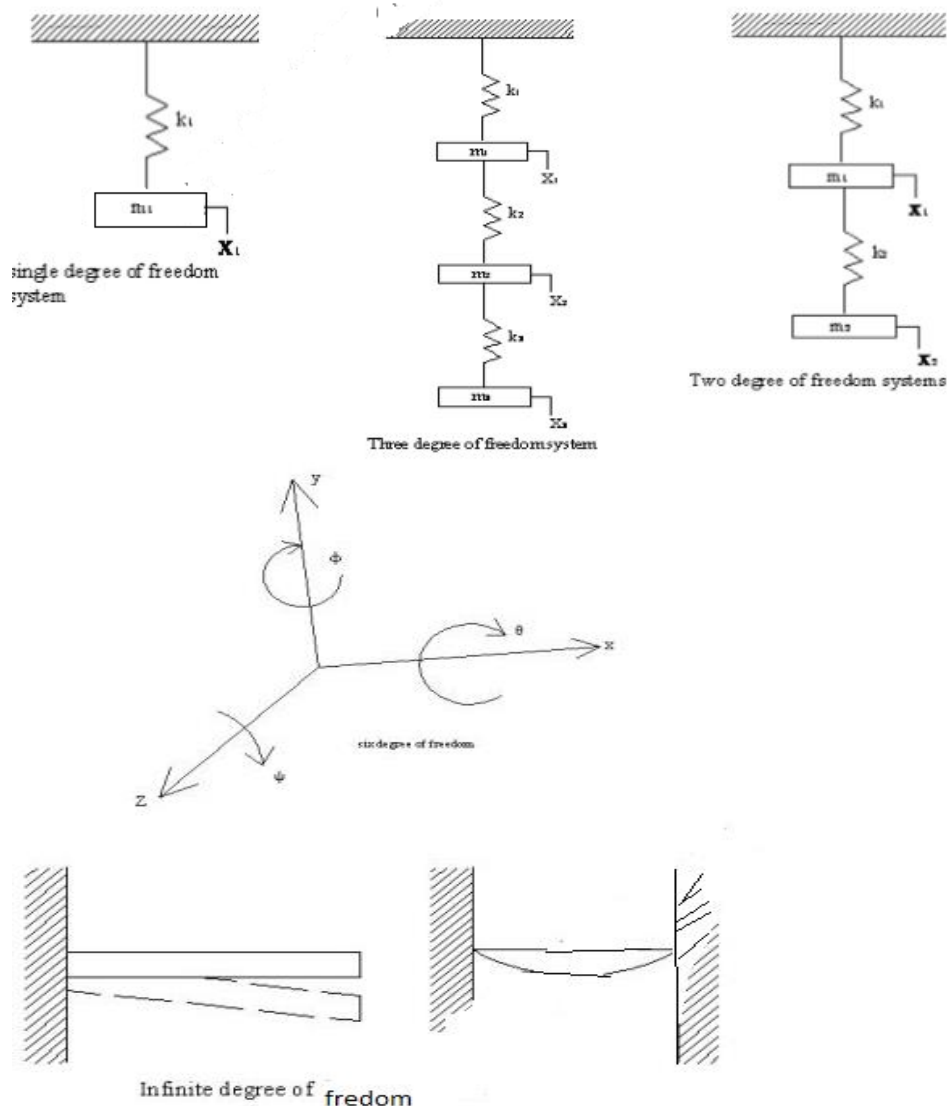
### 1.2. Methods to reduce vibrations

Elimination or reduction of the undesirable vibrations can be obtained by one or more of the following methods

1. Removing the cause of vibrations
2. Putting in screens if noise is the only objection
3. Resting the machinery in proper type of isolators
4. Shock absorbers.
5. Dynamic vibration absorbers

**Vibration terminology:**

- 1) **Periodic motion:** A motion which repeats itself after equal intervals of time is known as periodic motion. Any periodic motion can be represented by function  $x(t)$  in the period  $T$ . the function  $x(t)$  is called periodic function
- 2) **Time period:** - Time taken to complete one cycle is called time period.
- 3) **Frequency:** - The number of cycles per unit time is known as frequency.
- 4) **Natural frequency:** - When no external force acts on the system after giving it an initial displacement, the body vibrates. These vibrations are called free vibrations and their frequency as natural frequency. It is expressed in c/s or hertz
- 5) **Amplitude:** - The max displacement of a vibrating body from its equilibrium position is called amplitude.
- 6) **Fundamental mode of vibration:** - The fundamental mode of vibration of a system is the mode having the lowest natural frequency.
- 7) **Resonance:** - When the frequency of external excitation is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This concept is known as resonance.
- 8) **Mechanical systems:** - The systems consisting of mass stiffness and damping are known as mechanical systems.
- 9) **Continuous and discrete systems:** - Most of mechanical systems include elastic members which have infinite number of degree of freedom. Such systems are called continuous systems. Continuous systems are also known as distributed systems. Ex. Cantilever, Simply supported beam etc. Systems with finite number of degrees of freedom are called discrete or lumped systems.
- 10) **Degree of freedom:** - The minimum no of independent co-ordinates required specifying the motion of a system at any instant is known as degree of freedom of the system. Thus a free particle undergoing general motion in space will have three degree of freedom, while a rigid body will have six degree of freedom i.e. three components of position and three angles defining its orientation. Furthermore a continuous body will require an infinite number of co-ordinates to describe its motion; hence its degree of freedom must be infinite.
- 11) **Simple harmonic motion (SHM)** A periodic motion of a particle whose acceleration is always directed towards the mean position and is proportional to its distance from the mean position is known as SHM. It may also be defined as the motion of a projection of a particle moving round a circle with uniform angular velocity, on a diameter.
- 12) **Phase difference** It is the angle between two rotating vectors representing simple harmonic motion of the same frequency



**2.2.2 Classification of vibrations** Mechanical vibrations may broadly be classified the following types

- 1) Free and forced vibration
- 2) Linear and nonlinear vibration
- 3) Damped and un damped vibration
- 4) Deterministic and random vibration
- 5) Longitudinal, transverse and torsional vibration
- 6) Transient vibration

### 1. Free and Forced vibration

Free vibration takes place when system oscillates under the action of forces inherent in the system itself and when external impressed forces are absent. The system under free vibration will vibrate at one or more of its natural frequencies. Vibration that takes place under the excitation of external forces is called forced vibration. When the excitation is oscillating the



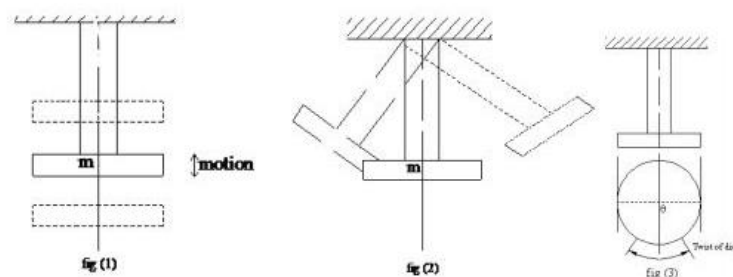
system is forced to vibrate at the excitation frequency. If the frequency of excitation coincides with one of the natural frequency of the system, a condition of resonance is encountered, and dangerously large oscillations may result.

**2) Linear and Non-linear vibration** If in a vibrating system mass, spring and damper behave in a linear manner, the vibrations caused are known as linear in nature. Linear vibrations are governed by linear differential equations. They follow law of superposition. On the other hand, if any of the basic components of a vibrating system behaves non-linearly, the vibration is called non-linear. Linear vibration becomes non-linear for very large amplitude of vibration. It does not follow the law of superpositions.

**3) Damped and Undamped vibration** If the vibrating system has a damper, the motion of the system will be opposed by it and the energy of the system will be dissipated in friction. This type of vibration is called damped vibration. The system having no damper is known as undamped vibration

**4) Deterministic and Random vibration** If in the vibrating system the amount of external excitation is known in magnitude, it causes deterministic vibration. Contrary to it the non-deterministic vibrations are known as random vibrations.

**5) Longitudinal, Transverse and Torsional vibration** Fig represents a body of mass 'm' carried on one end of a weightless spindle, the other end being fixed. If the mass moves up and down parallel to the spindle and it is said to execute longitudinal vibrations as shown in fig (1).



When the particles of the body or spindle move approximately perpendicular to the axis of the spindle as shown in fig (2) the vibrations so caused are known as transverse vibrations if the spindle gets alternately twisted and untwisted on account of vibrating motion of the suspended disc, it is said to be undergoing torsional vibrations as shown in fig(3).

**6) Transient Vibration** In ideal system the free vibrations continue indefinitely as there is no damping. The amplitude of vibration decays continuously because of damping (in a real system) and vanishes ultimately. Such vibration in a real system is called transient vibration

## ILLUSTRATIONS

### 1. What do you by vibration? Explain the causes of Vibration.

Ans) Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. The oscillations may be periodic such as the motion of a pendulum or random such as the movement of a tire on a gravel road.

**Causes of vibration:** - The main causes of vibration are:-

**Unbalanced forces in the machine.** These forces are produced from within the machine itself because of non-uniform material distribution in a rotating machine element.

**Dry friction between the two mating surfaces:** This is what known as self-excited vibration. **External excitations.** These excitations may be periodic, random or of the nature of an impact produced external to the vibrating system.

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### 2. Define the following:

#### a) Periodic Motion b) Time Period c) Frequency d) Amplitude

**ans) Periodic motion:** A motion which repeats itself after equal intervals of time is known as periodic motion. Any periodic motion can be represented by function  $x(t)$  in the period  $T$ . the function  $x(t)$  is called periodic function.

**Time period:** - Time taken to complete one cycle is called time period.

**Frequency:** - The number of cycles per unit time is known as frequency.

**Amplitude:** - The max displacement of a vibrating body from its equilibrium position is called amplitude

### 4. Explain any five types of vibrations, with examples.

#### Ans) 1. Free and Forced vibration

Free vibration takes place when system oscillates under the action of forces inherent in the system itself and when external impressed forces are absent. The system under free vibration will vibrate at one or more of its natural frequencies. Vibration that takes place under the

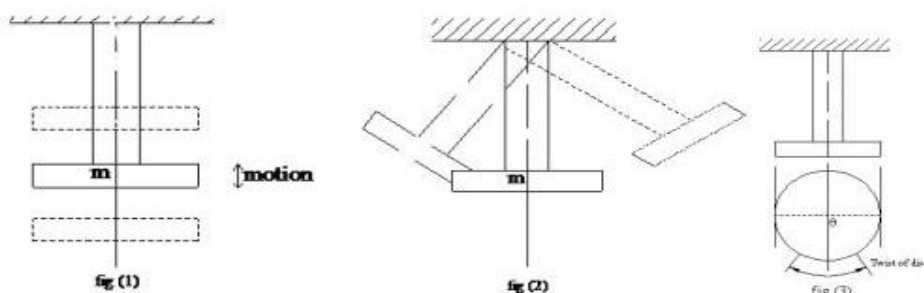
excitation of external forces is called forced vibration. When the excitation is oscillating the system is forced to vibrate at the excitation frequency. If the frequency of excitation coincides with one of the natural frequency of the system, a condition of resonance is encountered, and dangerously large oscillations may result.

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# MODEL QUESTIONS - I

4th sem Mech Engg

Subject code - TH 1

## Theory of Machine

F.M. 80

Time - 3 hr

Answer Any five question including Q 1 & 2

1. Answer all questions -

- (a) What is a kinematic link?
- (b) What do you mean by degrees of freedom?
- (c) What do you mean by limiting friction?
- (d) What is the function of clutch & brake?
- (e) What is initial tension in belt drive?
- (f) What is centrifugal tension?
- (g) What is the function of governor?
- (h) What do mean by coefficient of fluctuation of speed?
- (i) What do you mean by Dynamic Balancing of Rotating masses?
- (j) Define Amplitude, time period in vibration terms.

2. Answer any six questions 15x6

- (a) Write down difference between higher pair & lower pair? Explain briefly different types of lower pair.
- (b) Derive the length of open Belt drive.
- (c) With neat sketch define pitch circle, circular pitch, module, addendum and dedendum.

(d) With neat sketch explain the working of a centrifugal governor

(e) Explain working principle of epicyclic gear train.

(f) Explain the causes and Remedies of vibration

(g) What is the function of cam and follower? Explain different types of C.A.M.

3. What is four bar mechanism? state and explain different inversion of four bar mechanism

4. Derive the torque transmitted in pivot bearing considering the uniform wear and uniform pressure.

5. Find the width of the belt required to transmit 7.5 kW to a pulley 300 mm diameter, if the pulley makes 1600 rpm and the coefficient of friction between the belt and pulley is 0.22. Assume the angle of contact as  $210^\circ$  and maximum tension in the belt is not to exceed 8 N/mm width.

6. A loaded Porter Governor has four links each 250 mm long, two revolving masses of each 3 kg and a central load of mass 20 kg. All the links are attached to respective sleeves at radial distance of 40 mm from the axis of rotation. The masses revolve at a radius of 150 mm at minimum speed and a radius of 200 mm at maximum speed. Determine range of speed.



**ASSIGNMENT QUESTION**  
**OF**  
**THEORY OF MACHINE(4<sup>th</sup> sem Mech Egg)**

**Module 1 Simple mechanism**

1. Explain the term kinematic link.
2. Give the classification of kinematic link.
3. What is a machine? Giving example, differentiate between a machine and a structure.
4. What do you mean by kinematic pairs?
5. What do you mean by constrained motion?
6. Explain the terms: 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion of mechanism
7. Explain different types of kinematic pairs.
8. Explain Grubler's criterion for determining degree of freedom for mechanisms.
9. Sketch and describe the four bar chain mechanism and inversion of four bar chain mechanism.
10. Sketch and explain the various inversions of a slider crank chain
11. Sketch slider crank chain and its various inversions,
12. Sketch and describe the working of two different types of quick return mechanisms.  
Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.

**Module 2 Friction**

1. Explain the following:  
i) Limiting friction, (ii) Angle of friction, and  
iii) Coefficient of friction iv) Angle of Repose
2. Distinguish between brakes and dynamometers.
3. Describe the construction and operation of a prony brake or rope brake absorption dynamometer.
4. Derive an expression for the effort required to raise a load with a screw jack taking friction into consideration.
5. Neglecting collar friction, derive an expression for mechanical advantage of a square threaded screw moving in a nut, in terms of helix angle of the screw and friction angle
6. Deduce an expression for the friction torque on flat pivot bearing, assuming uniform pressure and wear.
7. Derive an expression for the friction torque for a flat collar bearing. Assume uniform intensity of pressure.
8. Derive from first principles an expression for the friction torque of a conical pivot assuming (i) Uniform pressure, and (ii) Uniform wear.
9. Describe with a neat sketch the working of a single plate friction clutch.

10. A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end footstep bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction
11. A conical pivot supports a load of 20 kN, the cone angle is  $120^\circ$  and the intensity of normal pressure is not to exceed  $0.3 \text{ N/mm}^2$ . The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.
12. A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine 90 r.p.m., find the power absorbed in friction at the thrust block, assuming 1. uniform pressure; and 2. Uniform wear.
13. A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed  $0.1 \text{ N/mm}^2$ . If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.

### **Module 3 Power Transmission**

1. What is velocity ratio of a belt drive?
2. Explain the phenomena of 'slip' and 'creep' in a belt drive
3. What is centrifugal tension in a belt
4. Obtain an expression for the length of a belt in 1. an open belt drive ; and 2. a cross belt drive.
5. For a flat belt, prove that 1
 
$$T_1/T_2 = e^{\mu\theta}$$
 Where  
 $T_1$  = Tension in the tight side of the belt,  
 $T_2$  = Tension in the slack side of the belt  
 $\mu$  = Coefficient of friction between the belt and the pulley, and  
 $\theta$  = Angle of contact between the belt and the pulley (in radians.)
6. Derive the condition for transmitting the maximum power in a flat belt drive.
7. Derive the centrifugal tension in a flat belt.
8. Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?
9. A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is  $165^\circ$  and the coefficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa,

density of leather 1 Mg/m<sup>3</sup> and thickness of belt 10 mm, determine the width of the belt taking centrifugal tension into account.

10. Explain the terms : (i) Module, (ii) Pressure angle, and (iii) Addendum
11. Explain briefly simple, compound, and reverted gear trains
12. Explain with neat sketch epicyclic gear trains.
13. How the velocity ratio of epicyclic gear train is obtained by tabular method?
14. In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B?

### **Module 4 Governors and Flywheel**

1. What is the function of a governor?
2. How does governor differ from that of a flywheel?
3. State the different types of governors. What is the difference between centrifugal and inertia type governors?
4. Define and explain the following terms relating to governors :
5. Stability, 2. Sensitiveness, 3. Isochronism and 4. Hunting.
6. Define 'effort' and 'power' of a Porter governor.
7. Explain the term height of the governor. Derive an expression for the height in the case of a Watt governor.
8. Define the terms 'coefficient of fluctuation of speed
9. What is the function of a flywheel? How does it differ from that of a governor
10. Prove that the max fluctuation of energy  

$$\Delta E = 2 E C_s$$

Where  $E$  = Mean kinetic energy of the flywheel, and  $C_s$  = Coefficient of fluctuation of speed.
11. A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor
12. A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor
13. A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are

150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor

14. A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : 1. loads on the spring at the lowest and the highest equilibrium speeds, and 2. stiffness of the spring

### **Module 5 Balancing of Machine**

1. Define the terms 'static balancing' and 'dynamic balancing'.
2. Four masses  $m_1$ ,  $m_2$ ,  $m_3$  and  $m_4$  are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively. and the angles between successive masses are  $45^\circ$ ,  $75^\circ$  and  $135^\circ$ . Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.
3. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B  $45^\circ$ , B to C  $70^\circ$  and C to D  $120^\circ$ . The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

### **Module 6 Vibration of machine parts**

1. What are the causes and effects of vibrations?
2. Define, in short, free vibrations, forced vibrations and damped vibrations.
3. What are longitudinal, transverse and torsional free vibrations?



